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FLUID FLOW IN PETROLEUM RESERVOIRS

I. – The Kozeny Paradox

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ABSTRACT

This study was undertaken to make an analytical check of the Kozeny equation as a description of fluid flow in porous media, particularly as it applies to the mixture-flow phenomena in petroleum reservoirs where the anisotropies of sedimentary rock and the hysteresees of capillarity introduce many complications in analysis.

My analysis shows that the Kozeny equation predicts an increase in permeability when a decrease would be expected, as for example the decrease in permeability that would seem to result from the consolidation and cementing of an unconsolidated sand. Perhaps this is an apparent rather than a real paradox; but nevertheless the paradox should be resolved before further use is made of the Kozeny equation to describe fluid flow in petroleum reservoirs.

INTRODUCTION

Numerous authors have stated that for certain situations the Kozeny equation adequately describes mixture flow in porous media such as petroleum reservoirs. Although some other authors have doubted the value of these applications, especially if the pore structure is non-uniform and anisotropic, no one heretofore has proved there is a gross error in using a Kozeny equation of the following type to describe relative permeabilities of the wetting and nonwetting phases for clean, well sorted sands:

$$K_r = (S) (A_1/A)^2 (t_1/t) \quad (1)$$

In Equation (1) K_r is the fractional relative permeability offered by the porous media to the flow of a fluid phase that has a fractional saturation of S . A and t are, respectively, the specific surface area per unit pore volume and the Kozeny textural parameter descriptive of the boundary conditions of that part of the pore space filled with the fluid phase of interest; A_1 and t_1 are the related limiting values when $S = \text{unity}$.

The Kozeny equation also has been proposed as applicable to a description of homogeneous fluid flow through consolidated sandstones, at least when the pore sizes are uniform and the porous continuum is isotropic. Thus, it is sometimes stated that

$$K = f/A_1^2 t_1 \quad (2)$$

where f is the fractional porosity and K is the specific permeability.

The pros and cons of the postulates stated above have been adequately discussed by Wyllie (1952) and by Wyllie and his co-workers (1952, 1955). For present purposes, it will suffice to note that of all the terms in Equation (1),

only S can be determined with accuracy; moreover, there is the serious question as to whether or not A and t can be measured independently. Similarly, there is no wide agreement as to how A_1 and t_1 in Equations (1) and (2) should be measured, except for special cases such as packings of uniform-sized spheres. But the use of Equation (1), or Equation (2) for consolidated sandstones, can be discredited on more fundamental grounds.

If a random packing of unconsolidated, uniform-sized spheres is taken as the model, Equation (2) describes the details of homogeneous liquid flow in the viscous regime. In such a case, t_1 has the universal value of 5 regardless of sphere size, and A_1 is simply inversely proportional to sphere size so that permeability is proportional to the square of the sphere radius times the appropriate porosity function.

For simplicity, and without requiring proof, it also can be said that Equation (2) at least approximately describes fluid flow through regular packings of uniform-sized spheres. In such a case, values of t_1 ranging from 4 to 5.7 can be calculated for cubic and rhombohedral packings, respectively, by using the data of Pirson (1947) according to the formation factor method discussed by Wyllie and Spangler (1952).

APPLICATION OF THE KOZENY EQUATION

The consequences of Equation (1) can now be examined for various types of packings of uniform-sized spheres, at least for the condition of flow of a non-wetting phase when the wetting phase is distributed uniformly as pendular rings. Smith et al. (1930) and Rose (1957a) have presented independent but equivalent expressions for the saturation of pendular wetting phase as a function of porosity, such as would be observed in packings of uniform-sized spheres, which have the form

$$S_w = v N n / 2$$

where $v = (2 \pi R^3) (F_1 - \theta) =$ volume of each pendular ring.

$N = [(1-f) / f] [3 / (4 \pi R^3)] =$ number of spheres per unit pore volume.

$n = \frac{(1 + 1.828 x)}{(1 + 0.414 x)} 6 =$ number of sphere contacts per sphere.

$x = (0.476 - f) / 0.217 =$ fraction of spheres packed in rhombohedral array [N.B. $(1-x)$ is the number of spheres packed in cubic array].

(3)

Rose (1957a) also has given the surface area of each pendular ring as

$$a = (4 \pi R^2) (F_2 - \theta)$$

Hence: $A_i = \frac{N a n}{2} =$ interfacial surface area between wetting and non-wetting fluids per unit pore volume.

$$A_{sn} = 2\pi R^2 N [2 - n(1 - \cos \theta)] = \text{specific surface area between nonwetting phase and pore walls.}$$

$$A_i = 3 \frac{(1-f)}{(fR)} = 4\pi R^2 N = \text{specific surface area of all pore surfaces.}$$

$$A_n = A_i + A_{sn} = \text{specific surface area of all surfaces bounding the nonwetting phase when the saturation is } (1 - S_w).$$

$$\lim_{S_w \rightarrow 0} A_n = A_i \quad (4)$$

Similarly:

$$A_{sw} = 2\pi R^2 N n(1 - \cos \theta) = \text{specific surface area between wetting phase and pore walls.}$$

$$A_w = A_i + A_{sw} = \text{specific surface area of all surfaces bounding the wetting phase when the saturation is } S_w. \quad (5)$$

In the above, θ is the angle subtending the line joining the sphere center with the contact point of the pendular ring on the surface of the sphere, and the line joining the centers of adjacent spheres. Thus $F_1\theta = \{2 - 2\cos\theta - \tan\theta [2\sin\theta - \tan\theta + (90^\circ - \theta)(1 - 1/\cos\theta)^2]\}$ and $F_2\theta = [(90^\circ - \theta)\tan\theta - (1 - \cos\theta)] [(1 - \cos\theta)/\cos\theta]$.

These functions are plotted in figure 1 for a cubic packing (that is, $x = 0$), and in figure 2 for rhombohedral packing (that is, $x = 1$). Also indicated are the various specific surface functions for conditions of funicular saturation, which were calculated by making the slightly incorrect assumption that the nonwetting phase is uniformly distributed as insular globules.

The discontinuities in the curves of figures 1 and 2 result from the imposed restrictions that: a) the insular globules of nonwetting phase can never be larger than the largest pore openings (namely, $0.366R$ and $0.414R$ for rhombohedral packing and $0.732R$ for cubic packing), and b) the pendular rings at any grain contact cannot exceed the volume where coalescence between contiguous rings will occur (namely, θ has a maximum value of 30° for rhombohedral packing and 45° for cubic packing).

Although Scott and Rose (1953) discussed the various specific surface area functions for a hypothetical capillary tube system, figures 1 and 2 appear to present the only extant information applicable to real, albeit idealized, porous media. It should be possible, therefore, to calculate the relative permeability function by Equation (1) upon assigning values to (t_1/t) versus saturation.

For relative permeabilities of the wetting phase, Wyllie and Rose (1950) gave the evidently valid approximation:

$$(t_1/t)_w^n = (S_w I)^{-2} \quad (6)$$

where I is the resistivity index.

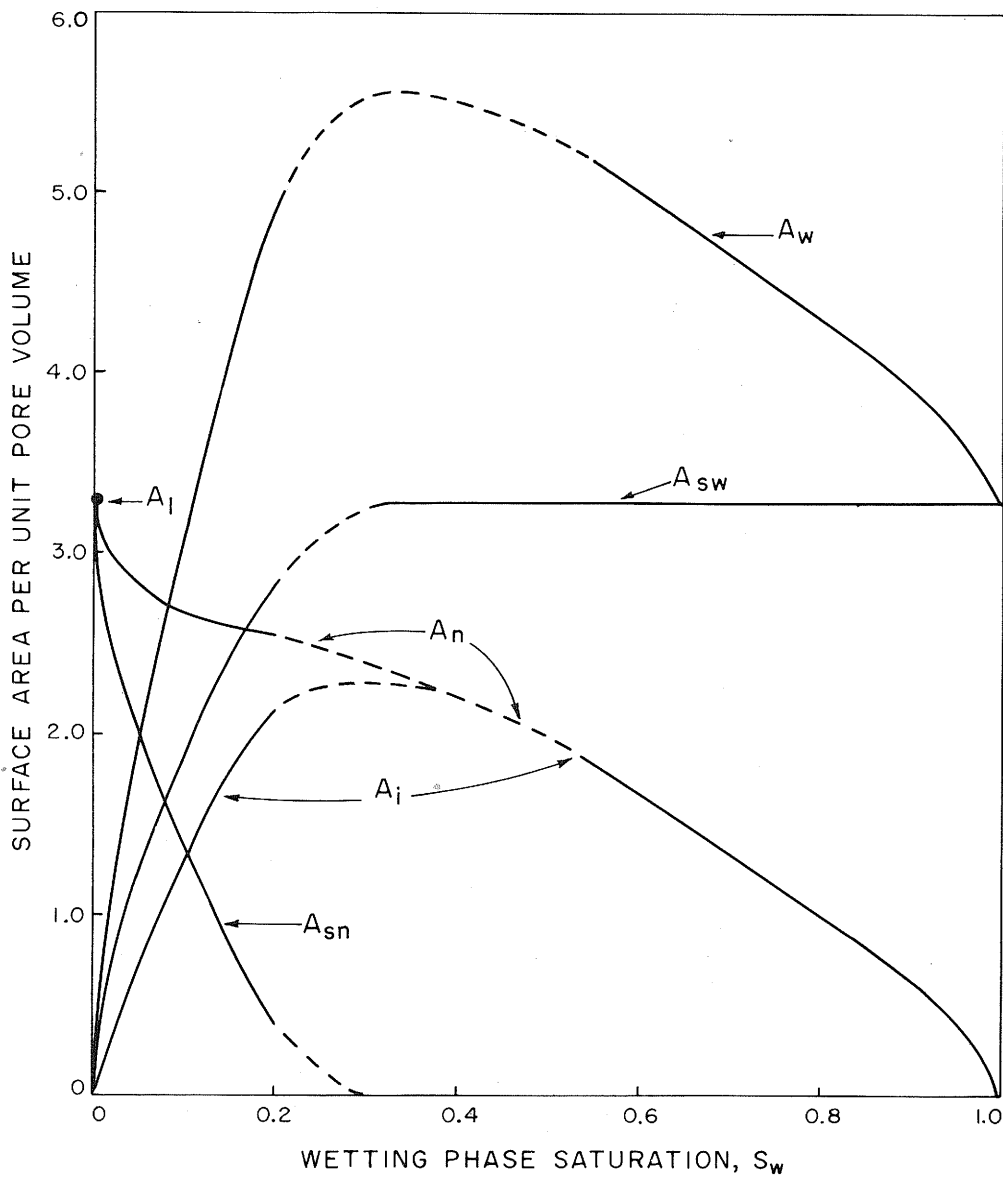


Fig. 1. - Specific surface area functions for cubic packing.

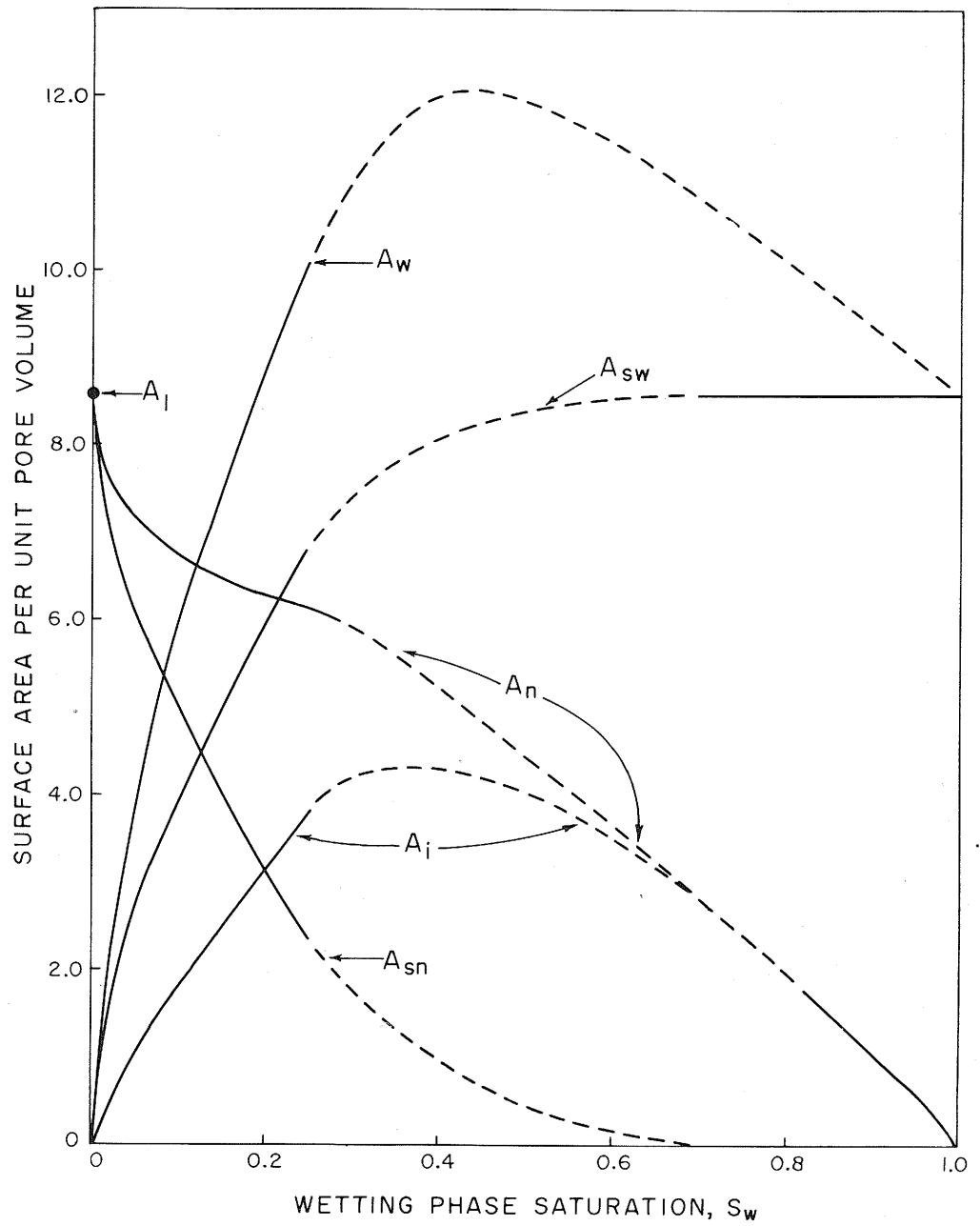


Fig. 2. - Specific surface area functions for rhombohedral packing.

For relative permeabilities of the nonwetting phase no such simple expression for $(t_1/t)_n$ is known, but it is likely that this ratio is unity (or slightly greater than unity) for packings of uniform-sized spheres, at least when the wetting phase is distributed as pendular rings.

The above postulations and propositions make it possible to examine the consequences of Equation (1). Assuming $(t_1/t)_n$ is unity and considering only saturation conditions of the pendular wetting phase, the values shown in table 1 are calculated.

Table 1 - Solution of Equation 1 for Cubical and Rhombohedral Packings

Cubic packing			Rhombohedral packing		
θ	S_w	K_{rn}	θ	S_w	K_{rn}
0°	0	1.00	0°	0	1.00
15	0.005	1.14	5	0.001	1.03
20	0.012	1.20	10	0.006	1.12
25	0.032	1.31	15	0.022	1.44
30	0.051	1.36	20	0.059	1.46
35	0.097	1.36	25	0.138	1.48
40	0.131	1.38	30	0.248	1.44
45	0.196	1.32			

The tabulation points to a paradoxical situation, for values greater than unity for relative permeability of the nonwetting phase are neither expected from theory (Scott and Rose, 1953) nor shown by data obtained on well sorted sand packings (Leverett, 1939), nevertheless they are indicated by Equation (1) which in other respects appears to be a valid relationship. The disparity is actually worse than that suggested by the values in table 1 because $(t_1/t)_n$ ratios are probably actually greater than unity. Also, if Yuster's consideration of transfer of viscous forces at fluid-fluid interfaces (Yuster, 1951) has any meaning, the tabulated relative permeability values should be increased further in consequence of the finite fluid-fluid interfacial area which partially bounds the nonwetting phase (and which would result in a slippage).

Continuing the argument, table 1 shows that effective permeabilities are increased by more than 10 percent even when less than 0.5 percent of the pore space has been filled by locating another phase at the points of sphere contact. If the other phase is thought of as a cementing solid, it would appear that Equation (2) no longer is a valid expression for fluid flow, even through only slightly consolidated porous media. The remarkable thing is that this appears to be true for the uniform particle size systems for reasons which would assume still greater importance if particle size variations are introduced, and if degree of cementation is increased.

Hence, it can be concluded that previous workers were wrong when they tacitly assumed that Kozeny type equations are descriptive of mixture-flow phenomena and of homogeneous fluid flow through consolidated porous media. Heretofore, it has been argued only that it is difficult to measure independently

the various terms of Equations (1) and (2) in order to test and prove their applicability. Apparently, however, there are even more fundamental objections. Specifically, it is reasonable to conclude that in the case of certain heterogeneous and anisotropic porous media (for example, vuggy and fractured limestones, cemented sandstones with cul-de-sac pores), the surface area boundaries and pore "tortuosities" are not related to permeability as described by the Kozeny equation; and evidently this is also true for more uniform and homogeneous systems such as consolidated packings of well-sorted spheres.

One possible explanation, suggested in part by Equation (4), can be based on analysis of a model system composed of sphere particles of many sizes. In such a case, the N and n factors would both increase, with the consequence that A_n [namely, the "A" term in Equation (1)] would increase rapidly and might exceed the value of A_1 . Also, as small spheres filled the space between large spheres, the $(t_1/t)_n$ ratio would decrease rapidly as the saturation of the wetting phase increased (Rose, 1957b). Both effects would tend to make values for relative permeability of the nonwetting phase, as calculated by Equation (1), more in accord with experimental results and theoretical expectations; but whether or not the subject paradox would be resolved thereby remains uncertain at the moment.

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