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Hydraulic Fracture Theory

Part II. – Fracture Orientation and Possibility of Fracture Control

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ABSTRACT

This study takes up the problem of hydraulic fracture mechanics, orientation of fractures, and whether their control is possible. In Part I, some theories on the mechanics of materials were adapted for use in dealing with mechanical problems of hydraulic fracture and also to help in describing conditions of stress in porous sediments.

Part II summarizes some current notions of hydraulic fracture mechanics, including my own view. It sets forth sample problems based on the theory developed in Part I.

Hydraulic fracture orientation and distribution are controlled by the condition of stress underground. The horizontal stress underground is altered by the pore pressure. Thus the magnitude of the pore pressure may influence the orientation and distribution of hydraulic fractures.

INTRODUCTION

It is generally agreed that the compressive stress in the rock at the time of fracture will tend to control the orientation of the fracture. Although numerous other factors may enter into the problem the compressive stresses are probably a dominant influence.

Using this major premise, together with the theory stated in Part I of this report, sample problems dealing with hydraulic fracture mechanics are now presented. A general discussion of ideas in the current literature regarding fracture orientation also is included here.

The third part of this report, now in preparation, will deal with laboratory experiments suggested by the theoretical studies presented in Parts I and II.

Definitions and Postulates

The following terms are commonly used in the discussion of hydraulic fracturing. The symbols are defined in Part I.

1) The overburden pressure, σ_z, is the total vertical compressive stress, approximately equal to the average specific weight of overlying sediment times the depth.

2) Treatment pressure, P_i, is the bottom hole pressure during the extension of a fracture.

3) Breakdown pressure, P_i , is the bottom hole pressure at the instant before fracture initiation. This may be equal to or greater than the treatment pressure.

The following is a list of postulates to be used in the next section. Some of these are given little defense and are submitted to the reader as highly reasonable premises on which there is general agreement in the literature:

a) The initiation of a fracture according to (40), above, requires that

$$P \approx \sigma_t + \sigma_n$$

where P is the pore fluid pressure, σ_t is the tensile strength, and σ_n is the compressive stress normal to the plane of impending fracture. If the tensile strength of the rock is negligible or zero, then, according to (37)

$$P = \sigma_n \tag{42}$$

is a condition of impending fracture.

b) The propagation in a brittle material of a crack due to internal pressure is accompanied by a high stress concentration at the edge of the crack, so that only a small fraction of the tensile strength of the material, small in itself, is effective in resisting the propagation of the fracture. Therefore, the treatment pressure, P_i, necessary to extend a fracture exceeds by a negligible amount the pressure necessary to hold open the fracture, or

$$P_i = \sigma_n \tag{43}$$

c) When the control of joint systems and planes of weakness on the course of the fracture are taken into account, the displacement of the fracture faces will tend to be in the direction of the least principal compressive stress whereas the fracture may follow a tortuous path whose general course would tend to be normal to the least stress.

d) Local stress conditions may be created by the shape of the well bore, fluid pressure gradients, or plastic deformation before the initiation of a fracture. Such local stress fields may influence the orientation of the fracture and the breakdown pressure. However, after the fracture has been initiated, the presence of the well bore and the local conditions around the well bore are of negligible influence and the pressure of the fluid extending the fracture will be equal to the regional stress normal to the displacement of the fracture.

Evidence of Fracture Orientation from Treatment Pressures and Regional Faulting

Hubbert and Willis (1957) point out that the orientation of the least compressive stress may be inferred from the faulting of a region.

If it appears that the observed faulting has been active up to the present, it is likely that the stress condition is close to failure. The condition of failure is approximated by Hubbert and Willis by means of the Mohr-Coulomb construction (fig. 8).

On the premise that hydraulic fractures are developed normal to the least compressive stress for the region, the orientation of fractures are then known on the basis of faulting.

The most common type of faulting in sedimentary basins is normal faulting, in which case the least normal stress is horizontal and the greatest stress is vertical. McGuire et al. (1954) pointed to normal faulting on the Gulf Coast as evidence that the least compressive stress was horizontal and because treatment pressures were less than would support the overburden they concluded that fractures were vertical.

If a strong parallelism of normal faulting exists, then, according to Hubbert and Willis, hydraulic fractures should be vertical and parallel also. If this is the case it should be possible to establish line drive waterfloods in some areas by connecting input wells with vertical fractures, an interesting possibility.

Hubbert and Willis' conclusions were based heavily on the assumption that horizontal fractures require treatment pressures equaling the total overburden load. This assumption was also made by McGuire et al. (1954) and

Walker (1949). The following argument is based on this assumption: Treatment pressures are consistently less than overburden pressures. The treatment pressures must be greater than overburden pressure in order to induce a horizontal fracture. Therefore, fractures are consistently vertical.

Those who take the contrary view will argue that horizontal fractures can be induced with pressures less than the average overburden pressure, because the overburden pressure is not evenly distributed and, therefore, horizontal fractures might be induced along paths of least overburden load. This mechanism was suggested by Yuster and Calhoun (1945) to explain the low parting pressures obtained in waterflooding. The fractures at that time were generally assumed to be horizontal.

Certainly we must admit that a certain unevenness in the vertical stress distribution exists, as few things are uniform in nature; but it is difficult to believe that the strength of sedimentary rocks is sufficient to sustain such large scale unevenness in overburden pressure distribution through geologic time at depths of several thousand feet, as would be required to explain the deficit between the usual treatment pressures and overburden pressures. One may also question why such distribution should exist if it could exist. This view that the rocks are not strong enough was taken by McGuire et al. (1954), and the Russian workers Zeltov and Kristionovich (1955).

Another objection arising when uneven overburden pressure is used to explain low treatment pressures is the likelihood that a well will be drilled in an area of higher than average overburden pressure as often as in areas of low overburden pressure. Unequal overburden distribution might perhaps be important for shallow depths, which were the circumstances for which the idea was first suggested by Yuster and Calhoun (1945).

Proponents of the view that horizontal fractures are in fact produced with lower than overburden treatment pressures will point to gamma ray logs that give a high reading at a single spot in the hole after the injection of radioactive sand, thus indicating a horizontal fracture even though the treatment pressure was less than the average overburden pressure.

There are two (and perhaps more) ways that such evidence could be misleading. We may have a vertical fracture in which the radioactive sand has been overflushed away from the hole at all but a single point, or we may have a vertical fracture inclined to the hole at a small angle and intersecting the hole within a distance of a few feet. It is assumed that when we use the term vertical we are using it loosely and include fractures inclined some 30° to the well bore.

One evidence for the production of horizontal fractures at less than overburden pressure is the reported propagation of fractures from an input well to a producing well during waterflooding such as described by Yuster and Calhoun. This feat is easy to visualize with horizontal fractures, but difficult if vertical fractures are assumed. I am not certain of the ease with which this experiment can be duplicated, so for me the matter is unsettled.

Control of Fractures

In spite of the evidence that regional stresses control the orientation of hydraulic fractures, those who maintain that fractures may be controlled by well bore and fracture treatment conditions also have a case. The ability of the fluid to penetrate the rock, and boundary conditions such as the shape of the hole and the thickness of the bed, will influence the local stress pattern that develops before breakdown occurs, and may control the path of initial breakdown.

A great deal of significance has been attached to the degree of fluid penetration and its effect on orientation of hydraulic fractures.

An experimental study by Scott, Bearden, and Howard (1953) demonstrated rather graphically what the effect of fluid penetration may be. They fractured drill cores with both penetrating fluids and an oil-base mud whose effective filtration rate was assumed to be zero. These tests seemed to indicate that a penetrating fluid would rupture the core along the bedding planes and that a nonpenetrating fluid broke the rock along the axis of the bore hole. Largely on the basis of this evidence, Clark and Reynolds (1954) proposed a process, "Vertifrac", which consisted essentially of fracture treatment with a nonpenetrating fluid.

When a nonpenetrating fluid is used the shape of the hole clearly defines the shape of the pressurized region that delivers the parting force to the rock until breakdown occurs. Figure 13 illustrates a well bore to be fractured with a nonpenetrating fluid. The pressure of the injected fluid is confined to section A, by an expandable packer at the top and at the bottom by a gravel pack. The fluid in the hole cannot penetrate A significantly because of mud cake built on the rock walls. When the fluid pressure is applied, it is confined to a cylindrical region defined by these boundaries.

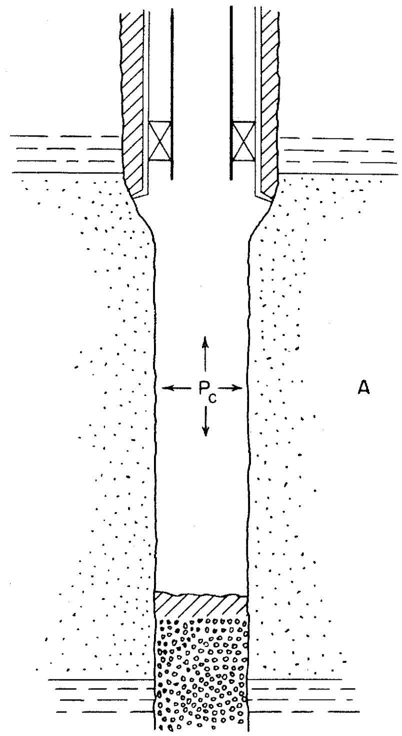

Fig. 13. - Fracturing with a "nonpenetrating fluid." The pressure is confined to the well bore by mud filter cake built on the walls.

What part of the axial thrust is delivered to the surrounding rock is uncertain. The upward thrust is transmitted diffusely to the rock walls by the cemented casing. The downward thrust is transmitted by shear forces distributed over the surface of contact between the wall of the hole and the gravel, and also against the bottom of the hole by the gravel. Such a diffuse distribution of the axial thrust leads one to suspect that the change in the axial stress within the section being considered is small compared to the change in the tangential stress due to the fluid pressure applied directly to the walls of the hole. In fact, it seems fairly certain that the initial breakdown must occur along the axis of the well bore. The example cited above roughly describes the "Vertifrac" process, which will be further dealt with in a sample problem.

A tool for the control of fracture orientation was reported recently by Gilbert et al. (1957). It consists of an assembly of shaped charges that in effect produce an oriented notch in the wall rock. When the fracturing pressure is applied, the fracture is alleged to propagate from the notch in a controlled orientation because of the oriented stress pattern that develops before breakdown.

If an oriented crack is produced by this tool, it is easy to believe that the fracturing fluid might enter and open a fracture in this plane in preference to a plane normal to the least stress along which no pre-existing fracture exists.

As it may be possible to control the fracture as it is first propagated from the well bore by a locally altered stress field, and as we recognize the tendency of the fracture to lie normal to the least principal stress, we are faced with the question of whether the fracture will continue in its "controlled" path or will align itself to the regional stress field as it is extended.

In my opinion, a fracture induced in a non-preferred orientation but following a tortuous path due to variously oriented planes of weakness will have a slight selectivity in choice of path that would gradually orient the general path taken by the fracture into the preferred plane. On the other hand, if the fracture is initiated along a single dominant plane of weakness, such as a bedding plane, it is reasonable that the fracture may continue in this plane in spite of a regional stress field that would favor vertical orientation.

In any case, fractures can be extended only by pressures greater than the compressive stress in the direction of fracture displacement. Therefore "controlled fractures" should be verified by the treatment pressures it took to produce them.

SAMPLE PROBLEMS

Fracturing with a non-penetrating fluid

The well bore subjected to internal fluid pressure, where the fluid is prevented from filtering out into the surrounding rock by a mud filter cake, has often been treated as a cylinder of infinite wall thickness subjected to internal and external pressure.

The Lamé formulae and special cases of the Lamé formulae follow:

$$\sigma_\theta = \frac{\sigma_{r_e} r_e^2 - P_i r_i^2 - \frac{r_i^2 r_e^2}{r^2}(P_i - \sigma_{r_e})}{r_e^2 - r_i^2} \qquad a$$

$$\sigma_r = \frac{\sigma_{r_e} r_e^2 - P_i r_i^2 + \frac{r_i^2 r_e^2}{r^2}(P_i - \sigma_{r_e})}{r_e^2 - r_i^2} \qquad b \qquad (44)$$

$$\sigma_z = \sigma_{z_o} = \text{constant} \qquad c$$

Letting $r_e \to \infty$ we obtain the special case of (44) for a cylinder of infinite wall thickness

$$\sigma_\theta = \sigma_{r_e} - \frac{r_i^2}{r^2}(P_i - \sigma_{r_e}) \qquad d$$

$$\sigma_r = \sigma_{r_e} + \frac{r_i^2}{r^2}(P_i - \sigma_{r_e}) \qquad e$$

$$\sigma_z = \sigma_{z_o} = \text{constant}$$

where r_i is the bore radius, P_i is the fluid pressure in the well bore, and σ_{r_e} is the regional horizontal stress. For simplicity we will assume that the two principal horizontal stresses are equal to σ_r. We will further assume that the tensile strength of the rock is negligible, so the condition of impending tensile failure will be $P = \sigma_n$, where P is the pore pressure and σ_n is the total stress normal to the plane of failure.

The problem will be to determine the bottom hole pressure, P_i, necessary for breakdown and the pressure necessary to extend the fracture in the vertical plane.

For breakdown to occur along the axis of the well bore, a necessary condition is $P = \sigma_\Theta$. Substituting P for σ_Θ in (44d) and letting r equal r_i gives us

$$P_i = 2\,\sigma_{r_e} - P$$

as the well bore breakdown pressure. But according to (41)

$$\sigma_{r_e} = \sigma_{r_o} + \left[\frac{JE}{1-\mu}\right] P$$

So

$$P_i = 2\sigma_{r_o} + 2\left[\frac{JE}{1-\mu}\right] P - P \qquad (45)$$

Let $\left[\frac{JE}{1-\mu}\right]$ equal .7, the value used previously for a porous sand. Then

$$P_i = 2\,\sigma_{r_o} + .4\,P$$

where σ_{r_o} is the regional horizontal stress at zero pore pressure.

Equation (45) indicates that for a porous sand the breakdown pressure will increase with increasing pore fluid pressure. For a hard sand $\frac{JE}{1-\mu} \approx 0$ and the breakdown pressure is

$$P_i = 2\sigma_{r_o} - P$$

After breakdown the system changes. The fluid pressure then acts within the fracture and according to postulate (d) the fluid pressure falls to the value of the regional compressive stress normal to the fracture, or simply $P_i = \sigma_r$. Letting $\frac{JE}{1-\mu} = .7$ and using equation (41) this becomes

$$P_i = \sigma_{r_o} + .7\,P$$

and for the hard sand, letting $\frac{JE}{1-\mu} = 0$

$$P_i = \sigma_{r_o} = \text{constant}$$

These results are plotted in figure 14 for arbitrary values of P and P_i, denoting either the treatment pressure or breakdown pressure, P, the pore fluid pressure and σ_{r_o}, the regional horizontal stress, when P equals zero.

The theory presented here governing breakdown pressure is not likely to have much practical application because actual breakdown pressures must be governed by many capricious unknowns such as the effective tensile strength of the rock imperfections in the hole, and possibly plastic yield of the rock around the hole.

The treatment pressure, on the other hand, has little to do with the hole condition (we are not taking fluid-flow resistance into account), and it is not dependent on the tensile strength of the rock, according to postulate b. It is therefore much more predictable.

Note in figure 14 that the treatment pressure for the soft sand rises steadily with .7 P . The treatment pressure for the hard sand is constant for $P < \sigma_{r_o}$ and increases with P for the interval $P > \sigma_{r_o}$, according to (37).

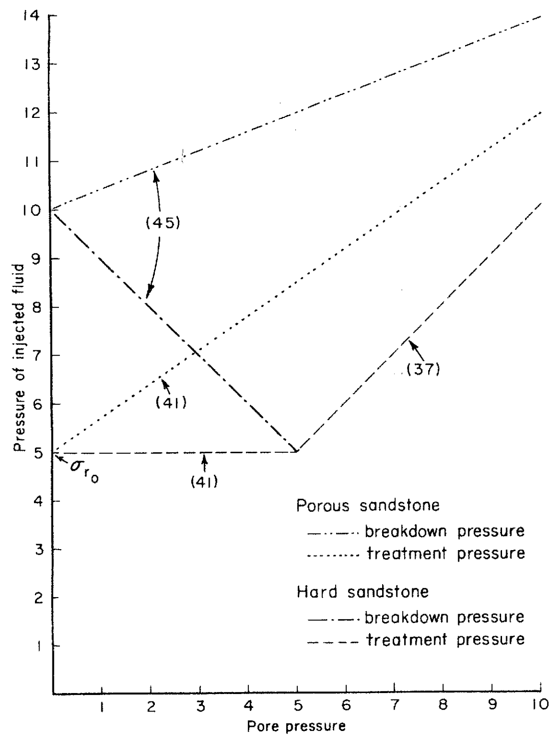

Fig. 14. - Treatment and breakdown pressures as a function of the pressure of the pore fluid pressure for the "non-penetrating fluid" case for a porous and a non-porous bed.

The value of σ_{r_o} used in figure 14 is the same for each sand. Its value in the general case would be different so the relative positions of the curves for the two sands have no significance.

"Thin bed" radial flow problem

Two cases of fracturing with a penetrating fluid will be considered. The first of these involves sustained injection of fluid through casing perforations into a thin bed of sand bounded by soft shales. Assume that in the shales $\sigma_\theta = \sigma_r = \sigma_z$ and in the sand $\sigma_{r_o} = \sigma_{\theta_o} < \sigma_z$.

When the fluid has been injected for an extended period of time, a pressure gradient will have developed that can be described approximately by the steady state radial flow equation

$$P = P_i + \frac{P_i - P_e}{\ln (r_i/r_e)} \ln (r/r_i) \tag{46}$$

This formula applies specifically to two dimensional radial flow from a cylindrical source but it also will give the pressure distribution around a cased and perforated well, except when r/r_i is small if the value of r_i used is the radius of an open hole well of equivalent conductivity.

The condition "thin bed" used to describe the problem means that the dimensionless group

$$T\left[J\,\frac{dP}{dr}\right]$$

is small in magnitude. T is the bed thickness and $J\,\frac{dP}{dr}$ is the gradient of the radial strain, assuming free expansion of the elements of the system. The bed in question is constrained between two semi-infinite regions. The restraining shear stresses, τ_r, delivered to the bed by these regions, are proportional to the bed thickness, T, and strain gradient, $\left[J\,\frac{dP}{dr}\right]$, which would result in the absence of restraint. Therefore, if T or $\left|\frac{dP}{dr}\right|$ became small, the bed may be considered entirely controlled by the regions above and below, and horizontal displacements would be prevented.

In the present problem, the shear stresses at the bed contact necessary to prevent horizontal movement would be of the order

$$\tau = \frac{1}{2}\,\frac{E}{1-\mu}\left[TJ\,\frac{dP}{dr}\right]$$

Letting $r_i = \frac{1}{2}$ ft., $r_e = 100$ ft., $\frac{JE}{1-\mu} = .7$, $(P_i - P_e) = 1{,}000$ psi, and T = 2 ft. Then, for r = 1 ft., $\tau \approx 133$ psi, and for r = 2 ft., $\tau \approx 66$ psi. So, except for perhaps very near the well bore, τ_r is small and horizontal restraint must be nearly complete.

A layer of paint on a block of steel subjected to thermal stresses would be a good analogy to the condition assumed to exist in this problem. It is safe to say that in such a case the areal distribution of the paint is controlled by the steel.

Horizontal restraint by adjacent rock layers establishes the condition that the components of horizontal strain due to the pore-pressure gradient, ϵ_θ^* and ϵ_r^*, equal zero. It also is known that $\sigma_z^* = 0$. These are the same conditions that gave us (41). Substituting (41) in (46) gives

$$\sigma_r = \sigma_\theta = \sigma_{r_o} + \frac{JE}{1-\mu}\left[P_i + \left(\frac{P_i - P_e}{\ln(r_i/r_e)}\right)\ln(r/r_i)\right] \qquad (47)$$

This equation gives the horizontal stress as a function of the radius, except perhaps within the region $r < 3r_i$ where the pressure gradient is uncertain and the assumption of zero horizontal strain questionable.

If P_i is increased very gradually, the local pore pressure is increased and along with it the horizontal stress, according to (47). As a result, the "treatment" pressure for vertical fractures is increased.

Eventually the condition that $P = \sigma_\theta$ is reached locally. Thenceforth further increases in P must be accompanied by an equal increase in σ_θ. But if $P = \sigma_\theta$, then $P_i > \sigma_\theta$, the condition of breakdown (42). If a vertical fracture does form it will terminate in the shales because in the shales $\sigma_\theta = \sigma_z > P_i$. If the sand is thin enough, openings of vertical fractures may be prevented by the horizontal restraint of the shale boundaries. Whether or not vertical fractures occur, it will still be possible to continue injection into the bed until $P \geq \sigma_z$, at which time a horizontal fracture is induced.

In summary, there seem to be three possible sequences resulting from sustained injection into a "thin bed".

1) when $\sigma_{r_o} < \sigma_z$ and σ_r increases according to (41) until $\sigma_r > \sigma_z$ and a horizontal fracture is formed;

2) when $\sigma_{r_o} < \sigma_z$ and σ_Θ increases according to (41) until $P = \sigma_r$. Then σ_r increases with P until a horizontal fracture is formed after $\sigma_r > \sigma_z$; or

3) when σ_r increases according to (41) until $P = \sigma_r$. A vertical fracture is induced but σ_r continues to increase as P is increased. Eventually $\sigma_r > \sigma_z$ and a horizontal fracture can be formed.

"Thick bed" radial flow problem

In contrast to the previous problem, imagine that fluid has been injected into a thick bed of sandstone for a short period of time over a long open-hole section. A fluid pressure gradient has been established in a relatively small cylindrical region around the hole.

The first part of the problem is the determination of the stress components σ_Θ^*, σ_r^*, and σ_z^* resulting from the pressure inside the well, P_i, a uniform pore pressure, P_e, and the regional stresses σ_z and σ_r. Two more sets of stress components will then be obtained, due to the pore pressure gradient between r_i and r_e which goes from $(P_i - P_e)$ at r_i to zero at r_e.

The solution to the first set of components is the same as that obtained in the first sample problem. Applying (44 c, d, and e)

$$\sigma_\Theta^* = \sigma_{r_e} - \frac{r_i^2}{r^2}(P_i - \sigma_{r_e})$$

$$\sigma_r^* = \sigma_{r_e} + \frac{r_i^2}{r^2}(P_i - \sigma_{r_e}) \qquad (48)$$

$$\sigma_z^* = \sigma_z$$

From (41)

$$\sigma_{r_e} = \sigma_{r_o} + \frac{J\,E}{1-\mu}P_e$$

Similarly

$$\sigma_{\Theta_e} = \sigma_{r_o} + \frac{J\,E}{1-\mu}P_e$$

Substituting in (48)

$$\sigma_\Theta^* = \sigma_{r_o} + \frac{J\,E}{1-\mu}P_e + \frac{r_i^2}{r^2}(\sigma_{r_o} + \frac{J\,E}{1-\mu}P_e - P_i)$$

$$\sigma_r^* = \sigma_{r_o} + \frac{J\,E}{1-\mu}P_e - \frac{r_i^2}{r^2}(\sigma_{r_o} + \frac{J\,E}{1-\mu}P_e - P_i) \qquad (49)$$

and

$$\sigma_z^* = \sigma_{z_o}$$

neglecting the uncertainties of the axial thrust of the fluid column in the well bore.

The second part of the solution is obtained by applying the analogy between the stresses due to pore pressure gradients and those due to unequal heating.

When a long circular cylinder is subjected to radial forces that are independent of z, the problem is one of plane strain in the region sufficiently removed from the ends of the cylinder.

Timoshenko (1934, p. 372) gives the solution to the thermal stresses in a long, thick-walled cylinder that are due to steady state radial heat flow. In applying the analogy we substitute P, the pore pressure, for T, the temperature, and J, the coefficient of pore pressure expansion for, α, the coefficient of thermal expansion. Lubinski (1954) applies this formula to the problem of radial flow in a thick porous cylinder. Once we have made these substitutions, Timoshenko"s solution applied to our problem becomes

$$\sigma_r^{**} = \frac{1}{2}\left[\frac{JE}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{\ln\frac{r_e}{r} + \frac{r_i^2}{(r_e^2 - r_i^2)}\ln\frac{r_e}{r_i}\left[1 - \frac{r_e^2}{r^2}\right]\right\}$$

$$\sigma_\Theta^{**} = \frac{1}{2}\left[\frac{JE}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{\ln\frac{r_e}{r} + \frac{r_i^2}{(r_e^2 - r_i^2)}\ln\frac{r_e}{r_i}\left[1 + \frac{r_e^2}{r^2}\right] - 1\right\} \quad (50)$$

$$\sigma_z^{**} = \frac{1}{2}\left[\frac{JE}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{2\ln\frac{r_e}{r} + \frac{2r_i^2}{(r_e^2 - r_i^2)}\ln\left[\frac{r_e}{r_i}\right] - 1\right\}$$

Letting $r = r_e$, the stress components at the outer surface of the cylinder are

$$\sigma_{r_e}^{**} = 0$$

$$\sigma_{\Theta_e}^{**} = \frac{1}{2}\left[\frac{JE}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{2\frac{r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right\} \quad (51)$$

$$\sigma_{z_e}^{**} = \sigma_{\Theta_e}^{**}$$

Timoshenko's equations (50) are applicable when the long, thick cylinder is free to expand and lengthen under the influence of thermal, or, in our case, pore-pressure gradients.

In the problem under consideration, lengthening of the cylinder is prevented in order to maintain the plane strain condition, at the same time maintaining continuity with the material outside r_e. In addition, lateral expansion is partly restrained.

The axial strain of the unrestrained cylinder, ϵ_z^{**}, is determined by the stresses at the outer surface, $\sigma_{z_e}^{**}$, $\sigma_{\Theta_e}^{**}$, and $\sigma_{r_e}^{**}$, by the application of Hooke's Law (26) and as $\sigma_{z_e}^{**} = \sigma_{\Theta_e}^{**}$ and $\sigma_{r_e}^{**} = 0$

$$\epsilon_{z_e}^{**} = \epsilon_{\Theta_e}^{**} = \frac{(1-\mu)}{E}\sigma_{z_e}^{**} \quad (52)$$

It is now necessary to find the set of stress components σ_z^{***}, $\sigma_{\Theta_e}^{***}$, and $\sigma_{r_e}^{***}$ that will satisfy the following conditions when imposed on the surface of the freely expanded cylinder:

1) an axial strain, $-\epsilon_z^{**}$, will result, restoring the cylinder to its original length;

2) a tangential strain, $\epsilon_{\Theta_e}^{***}$, will result such that $[\epsilon_{\Theta_e}^{**} + \epsilon_{\Theta_e}^{***}]$ is the

tangential strain at the bore of a thick cylinder representing the region outside r_e subjected to an internal pressure $\sigma_{r_e}{}^{**}$, and

3) $\epsilon_{\Theta e}{}^{***}$ is the tangential strain on the surface $r = r_e$ produced by the external pressure $\sigma_{r_e}{}^{***}$ and axial stress $\sigma_z{}^{***}$.

The first condition, $-\epsilon_z{}^{**} = \epsilon_z{}^{***}$, in terms of the stress components is

$$\frac{-(1-\mu)}{E}\,\sigma_{z_e}{}^{**} = \frac{\sigma_{z_e}{}^{***}}{E} - \frac{\mu}{E}\,(\sigma_{r_e}{}^{***} + \sigma_{\Theta_e}{}^{***}) \qquad (53)$$

The second condition is satisfied by

$$(\epsilon_{\Theta_e}{}^{**} + \epsilon_{\Theta_e}{}^{***}) = \frac{-(1+\mu)}{E}\,\sigma_{r_e}{}^{***} \qquad (54)$$

Where $(\epsilon_{\Theta_e}{}^{**} + \epsilon_{\Theta e}{}^{***})$ is the tangential strain at $r = r_e$ in the thick cylinder $r_e \leq r \leq \infty$ due to the internal pressure $\sigma_{r_e}{}^{***}$.

The final condition is merely a statement of Hooke's Law

$$\epsilon_{\Theta_e}{}^{***} = \frac{\sigma_{\Theta_e}{}^{***}}{E} - \frac{\mu}{E}\,(\sigma_{r_e}{}^{***} + \sigma_{z_e}{}^{***}) \qquad (55)$$

for the component of tangential strain, $\epsilon_{\Theta_e}{}^{***}$, resulting from stress components, $\sigma_{\Theta_e}{}^{***}$, $\sigma_{r_e}{}^{***}$, $\sigma_{z_e}{}^{***}$, acting on the surface of the cylinder $r_i \leq r \leq r_e$.

From Lamé's thick-cylinder formulae (44), the relation between the tangential stress, $\sigma_{\Theta_e}{}^{***}$, at the surface, $r = r_e$, due to an external pressure, $\sigma_{r_e}{}^{***}$, is

$$\sigma_{\Theta_e}{}^{***} = \sigma_{r_e}{}^{***}\left[\frac{r_e{}^2 + r_i{}^2}{r_e{}^2 - r_i{}^2}\right] \qquad (56)$$

From (52), (55), and (54)

$$\sigma_{z_e}{}^{**} = \frac{-\sigma_{\Theta_e}{}^{***}}{1-\mu} - \frac{\sigma_{r_e}{}^{***}}{1-\mu} + \frac{\mu\sigma_{z_e}{}^{***}}{1-\mu} \qquad (57)$$

From (56) and (57)

$$\sigma_{z_e}{}^{**} = -\frac{\left[1 + \frac{r_e{}^2 + r_i{}^2}{r_e{}^2 - r_i{}^2}\right]\sigma_{r_e}{}^{***}}{(1-\mu)} + \frac{\mu}{1-\mu}\,\sigma_{z_e}{}^{***} \qquad (58)$$

From (53) and (56)

$$\sigma_z{}^{**} = -\frac{\sigma_z{}^{***}}{1-\mu} + \frac{\mu}{1-\mu}\left[1 + \frac{r_e{}^2 + r_i{}^2}{r_e{}^2 - r_i{}^2}\right]\sigma_r{}^{***} \qquad (59)$$

From (58) and (59)

$$\sigma_{z_e}{}^{***} = \left[1 + \frac{r_e{}^2 + r_i{}^2}{r_e{}^2 - r_i{}^2}\right]\sigma_{r_e}{}^{***} \qquad (60)$$

and from (60) and (56)

$$\sigma_{z_e}^{***} = \left[\frac{r_e^2 - r_i^2}{r_e^2 + r_i^2} + 1 \right] \sigma_{\theta e}^{***} \qquad (61)$$

and substituting (60) and (61) in (57)

$$\sigma_{z_e}^{**} = -\sigma_{z_e}^{***} \qquad (62)$$

The components σ_z^{***}, σ_r^{***}, σ_θ^{***} due to the surface pressures, $\sigma_{z_e}^{***}$, and $\sigma_{r_e}^{***}$, are obtained as follows. To maintain the plane strain condition, $\sigma_{z_e}^{***}$ is applied uniformly across the top of the cylinder, so for $r_i \leq r \leq r_e$

$$\sigma_z^{***} = \sigma_{z_e}^{***} \qquad (63)$$

and for $r_e < r < \infty$

$$\sigma_z^{***} = 0. \qquad (64)$$

The radial stress, σ_r^{***}, for the region $r_e < r < \infty$, due to the internal pressure $\sigma_{r_e}^{***}$, applying the Lamé formula (44) is

$$\sigma_r^{***} = \frac{r_e^2}{r^2} \sigma_{r_e}^{***} \qquad (65)$$

Similarly, the tangential stress for $r_e < r < \infty$ is

$$\sigma_\theta^{***} = -\frac{r_e^2}{r^2} \sigma_{r_e}^{***} \qquad (66)$$

For the region $r_i < r < r_e$, again applying the Lamé formula

$$\sigma_r^{***} = \sigma_{r_e}^{***} \frac{r_e^2}{r_e^2 - r_i^2} \left[1 - \frac{r_i^2}{r^2} \right] \qquad (67)$$

and

$$\sigma_\theta^{***} = \sigma_{r_e}^{***} \frac{r_e^2}{r_e^2 - r_i^2} \left[1 + \frac{r_i^2}{r^2} \right] \qquad (68)$$

Summarizing these results, we have for the region $r_i < r < \infty$

$$\sigma_\theta^{*} = \left[1 + \frac{r_i^2}{r^2} \right] \left[\sigma_{r_o} + \left(\frac{J E}{1 - \mu} \right) P_e \right] - \frac{r_i^2}{r^2} P_i$$

$$\sigma_r^{*} = \left[1 - \frac{r_i^2}{r^2} \right] \left[\sigma_{r_o} + \left(\frac{J E}{1 - \mu} \right) P_e \right] + \frac{r_i^2}{r^2} P_i \qquad (49)$$

$$\sigma_z^{*} = \sigma_{z_o}$$

For the region $r_i \le r \le r_e$

$$\sigma_r^{**} = \frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\ln\frac{r_e}{r} + \frac{r_i^2}{(r_e^2 - r_i^2)}\ln\frac{r_e}{r_i}\left(1 - \frac{r_e^2}{r^2}\right)\right]$$

$$\sigma_\Theta^{**} = \frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\ln\frac{r_e}{r} + \frac{r_i^2}{(r_e^2 - r_i^2)}\ln\frac{r_e}{r_i}\left(1 + \frac{r_e^2}{r^2}\right) - 1\right] \quad (50)$$

$$\sigma_z^{**} = \frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[2\ln\frac{r_e}{r} + \frac{2r_i^2}{(r_e^2 - r_i^2)}\ln\frac{r_e}{r_i} - 1\right]$$

For the region $r_e < r < \infty$

$$\sigma_r^{**} = 0;\ \sigma_\Theta^{**} = 0;\ \sigma_z^{**} = 0$$

By substituting for $\sigma_{z_e}^{***}$ and $\sigma_{r_e}^{***}$ their equivalents in terms of $\sigma_{z_e}^{**}$ using (61) and (62), then substituting for $\sigma_{z_e}^{**}$ using (51), we obtain for the region $r_e < r < \infty$

$$\sigma_r^{***} = -\frac{1}{4}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\frac{2r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right]\frac{r_e^2 - r_i^2}{r^2}$$

$$\sigma_\Theta^{***} = \frac{1}{4}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\frac{2r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right]\frac{r_e^2 - r_i^2}{r^2} \quad (69)$$

$$\sigma_z^{***} = 0$$

and for the region $r_i < r < r_e$

$$\sigma_r^{***} = -\frac{1}{4}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\frac{2r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right]\left[1 - \frac{r_i^2}{r^2}\right]$$

$$\sigma_\Theta^{***} = -\frac{1}{4}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\frac{2r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right]\left[1 + \frac{r_i^2}{r^2}\right] \quad (70)$$

$$\sigma_z^{***} = -\frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left[\frac{2r_i^2}{r_e^2 - r_i^2}\ln\frac{r_e}{r_i} - 1\right]$$

By combining the components $\sigma_{r,\Theta,z}^{**}$ and $\sigma_{r,\Theta,z}^{***}$ for the region $r_i < r < r_e$, the stresses due to the fluid pressure gradient are

$$\sigma_r^{****} = \sigma_r^{**} + \sigma_r^{***} = \frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{\ln\frac{r_e}{r} - \frac{r_i^2}{r^2}\left[\ln\frac{r_e}{r_i} + \frac{1}{2}\right] + \frac{1}{2}\right\}$$

$$\sigma_\Theta^{****} = \sigma_\Theta^{**} + \sigma_\Theta^{***} = \frac{1}{2}\left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\left\{\ln\frac{r_e}{r} + \frac{r_i^2}{r^2}\left[\ln\frac{r_e}{r_i} + \frac{1}{2}\right] - \frac{1}{2}\right\}$$

$$\sigma_z^{****} = \sigma_z^{**} + \sigma_z^{***} = \left[\frac{J E}{1-\mu}\right]\left[\frac{P_i - P_e}{\ln(r_e/r_i)}\right]\ln\frac{r_e}{r}$$

equations (71)

In the region $r > r_e$ the components $\sigma_{r,\theta,z}^{**}$ are zero, so for this region the stresses $\sigma_{r,\theta,z}^{****}$ equal the components $\sigma_{r,\theta,z}^{***}$. The total stress components are given by

$$\sigma_r = \sigma_r^* + \sigma_r^{****}$$

$$\sigma_\theta = \sigma_\theta^* + \sigma_\theta^{****} \qquad (72)$$

$$\sigma_z = \sigma_z^{****}$$

and for region $r_e < r < \infty$

$$\sigma_r = \sigma_r^* + \sigma_r^{**}$$

$$\sigma_\theta = \sigma_\theta^* + \sigma_\theta^{**} \qquad (73)$$

$$\sigma_z = \sigma_{z_o}$$

Figure 15 shows a plot of the stresses around a well bore due to the steady-state flow of fluid from the well into the surrounding formation. The pressure in the porous stratum grades from the well-bore pressure P_i to the undisturbed formation pressure P_e at a distance r_e from the bore axis. The bore axis coincides with the vertical σ axis in figure 15.

The assumed set of conditions are shown in the upper right hand corner of the figure.

The components, $\sigma_{r,\theta,z}^*$, given by equations (49) are due to the well-bore pressure. The components, $\sigma_{r,\theta,z}^{****}$, given by equations (71) are due to the pore pressure gradients. The resultant total stress components, $\sigma_{r,\theta,z}$, are shown as solid lines.

DISCUSSION AND CONCLUSIONS

In the example given as a review of Part I, the relation for shear strength as a function of pore fluid pressure (36) was plotted on the diagram, figure 11. The shear criterion was not taken into consideration in later problems, because I considered that enough had been said in light of present knowledge. Future workers may want to investigate further and somehow obtain usable values for ϕ the angle of internal friction, appropriate to the application used here.

Another type of behavior that has been ignored here is the inelastic consolidation that may take place in a porous material upon reduction in pore pressure. This certainly is important in dealing with silts and clays, but the inelastic compression of consolidated sands is probably not too important within the range of pressures encountered in the oil fields. For discussions of this type of behavior the reader is referred to Terzaghi and Peck (1948) and Geertsma (1956).

I should like to put some emphasis on the phenomenon described in the last part of the review problem in Part I, the opening of vertical joints due to an increase in the value of the pore-fluid pressure after it has become equal to the least horizontal stress. It can be seen from figures 11 and 12 that the condition $P = \sigma_x$ is more likely to occur in a bed of low porosity.

The basic assumption that large masses of rock will not sustain an "effective" tensile stress is highly reasonable and the conditions necessary for the occurrence

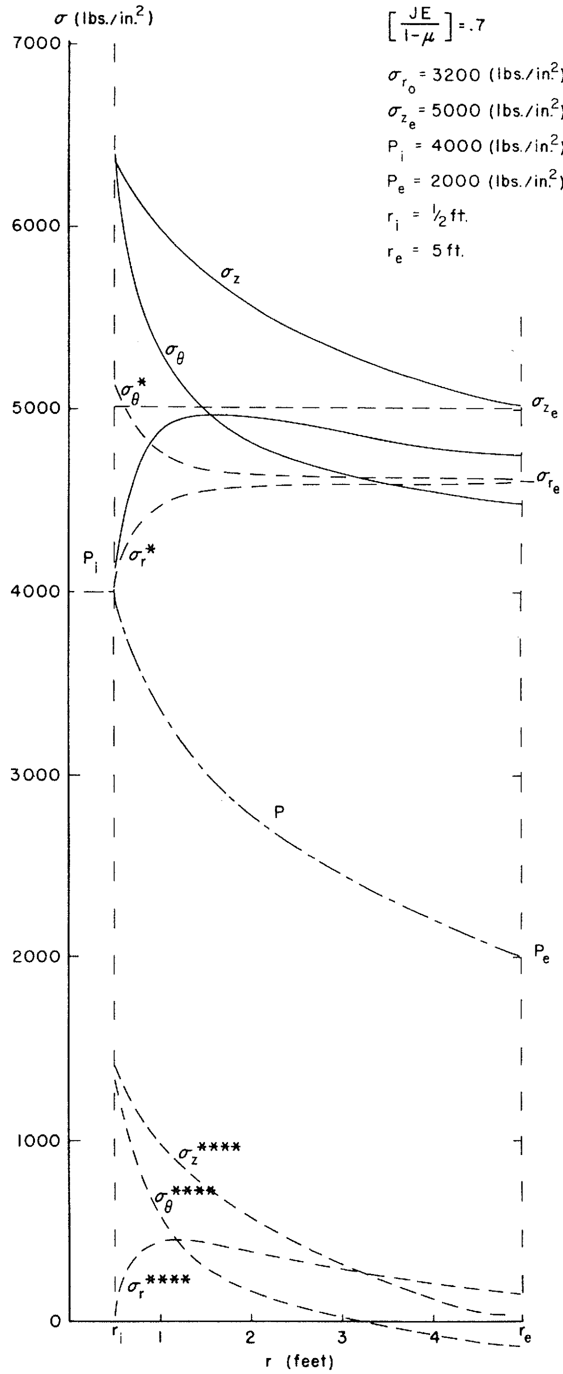

Fig. 15. - Stresses due to the steady-state radial flow of fluid from a well into a thick porous bed.

of the process, i.e., that one of the horizontal stresses be considerably less than the vertical stress, must be fairly widespread. It thus seems likely that the existence of this phenomenon may be fairly common.

A distinction must be made between a hydraulic fracture that is opened between two blocks of material actively forced apart by fluid pressure and a distribution of vertical fractures developed when the requisite pore pressure acts within a region. The latter fractures are analogous to shrinkage cracks that might develop in a plate of brittle material clamped between semi-infinite regions and then cooled.

The conditions for the production of these "shrinkage fractures" are about the same as those required for the induction of hydraulic fractures, i.e., condition (42).

Therefore, the production of these fractures generally should be accompanied or preceded by a hydraulic fracture extended from the injection well. However, in the situation described in the thin bed problem of Part II where a vertical fracture is confined within horizontal shale boundaries and the shales prevent horizontal displacements of any appreciable magnitude, it seems possible that an open vertical joint system might develop in the vicinity of the injection well unaccompanied by a major hydraulic fracture.

The main point is that these "shrinkage fractures" should exist, as the necessary condition for their existence is not unusual in the vicinity of an injection well.

The application of fracture theory in the field will be greatly facilitated by the technique of direct bottom-hole pressure measurement described in a paper by Godbey and Hodges at the October 1957 meeting of the AIME in which they give the results of bottom-hole pressure measurements during fracture treatment. Their investigation showed that the bottom-hole pressure during the extension of the fracture was fairly steady during treatment, irrespective of fluid properties and pumping rate, while the surface pressures varied widely.

The investigators also point to the great difficulty in obtaining the bottom-hole pressure by correcting the surface pressure for hydrostatic head and friction losses in the tubing because of the non-Newtonian flow characteristics and the changing density of the fluids.

The bottom-hole pressure during treatment is the key variable and its direct measurement should greatly facilitate the application of hydraulic fracture theory in the field.

Waterflood injection wells should be another important source of data for the study of pressure-parting phenomena. Good records of injection rate vs. input pressure may be kept, and the critical injection pressure resulting in pressure parting may be measured fairly accurately. Dickey and Andresen (1946) discuss in detail the technique for detecting pressure parting with input data.

These authors state, "Apparently the critical pressure is lower in the early life of a well, than after a considerable volume of water has been injected into the surrounding sand." This is consistent with the predicted increase in vertical parting pressure with formation fluid pressure given by equations (41) and (37).

I believe that horizontal fractures require pressures approximately equal to the overburden load. The treatment pressures required for the production of vertical fractures should vary with the pressure of the fluid in the formation in accordance with the theory presented here. Therefore, success in the control of fracture orientation by some special procedure should be substantiated or refuted by study of the bottom-hole treatment pressures required for fracture extension.

The problems given in Part II are intended to demonstrate the sorts of calculations that may be made with the theory presented, and specific conclusions should not be drawn from them.

In a general way it has been shown that treatment pressures for the production of vertical fractures may change with formation fluid pressure. Consequently, the tendency to fracture a given zone may change with formation fluid pressure. The fluid pressure distribution and bed thickness are important considerations. In general one can conclude that sustained injection before breakdown in a thin zone would be most conducive to horizontal fracture formation whereas a short period of injection in a thick bed, or zero fluid loss before breakdown, should lead to vertical fractures. It may be, however, that the orientation of fractures is dominated by the regional streses. If control of fractures is possible, the bottom-hole treatment pressure obtained should reflect the orientation of the fracture. If horizontal fractures are obtained, the bottom-hole treatment pressure should be about equal to the calculated overburden pressure. If the fracture is vertical the treatment pressure should be about equal to the least horizontal stress, which is normally less than the overburden pressure.

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