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STUDIES OF WATERFLOOD PERFORMANCE

IV. – Influence of Curtailments on Recovery

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ABSTRACT

This paper discusses the influence of curtailments on the efficiency of the waterflooding process. Curtailments are interruptions in production, such as shut-in periods, or controls put upon production, such as proration. Efficiency includes the idea of the cost of recovery, the rate of production, and the total (cumulative) recovery. Minimum costs are desired for economic reasons, high initial rates of recovery are favored for marginal operations, and maximum ultimate recovery is consistent with the conservation principle that a natural resource should never be lost.

My analysis is largely theoretical. It shows that the recovery process, including the waterflooding variant, is rate sensitive in a way that is related to the dominance of capillary forces; hence, curtailments will influence recovery. Examples are cited which show that high rates are advantageous in some cases whereas low are in others. This suggests that special analyses must be made of each field case if the appropriate maximum efficient and economic rates (the MER's) are to be determined. Certain guiding principles are suggested for this purpose.

INTRODUCTION

This paper deals with the specific problem of the influence of curtailments on oil recovery by waterflooding. Heretofore, numerous workers have considered this question but have reached little agreement. In fact, contradictory and inconclusive results appear to be the rule, as shown by the excellent summaries of opposing positions recently presented by Jordan et al. (1957) and Buckwalter et al. (1958).

Reservoir engineers always have been curious about the principles that govern the efficiency of the various oil recovery processes. Their central problem has been how to obtain an optimum balance between production and cost so as to realize maximum profits. This is an ever-challenging problem because it is never certain that the best answer has been found in any particular practical case.

Difficulty arises partly because the tools for inquiry are inadequate and limited. For example, mathematical and analog computational methods generally require the introduction of simplifying assumptions before solutions to a problem are possible. It may be necessary to approximate the reservoir size and shape by some simple and symmetrical geometric form, to neglect gravity and/or capillary forces, to disregard the differential mobilities of the displacing and displaced fluids, to represent transients as a succession of steady states, or to adopt other similar simplifications. One therefore hesitates to expect quantitative significance in the results.

The other techniques of problem solving are equally limited. For example, in core analysis methods the adequacy of scaling and the representativeness of the sample or samples are questionable. Field experiments and observations also may be inconclusive because of the inherent difficulty in making accurate, precise, and replicate field measurements, and because the field process is irreversibile, which means that comparisons can only be inferred. It also may be impractical to wait for completion of the field experiment, which might take tens of years (Jordan, 1958).

Such reasons explain why, even today, reservoir engineers know too little about how to propose systematically the MER for a particular field case, with full assurance that the advocated operating plan will yield the maximum efficient rate from which the maximum profit is realized.

Superimposed on these complications are others. The supply and demand situation may be such that it is convenient, perhaps mandatory, to operate particular properties above or below the MER. Similarly, workover of old wells and maintenance and installation of new facilities may necessitate interruptions in production of unpredictable but significant duration. Such conditions subordinate the need to know the factors controlling the efficiency of the recovery process.

The current interest in the influence of curtailments (such as proration, interrupted production due to shut-down days, etc.) on recovery has had simple origins. Operators seek to produce oil as fast as they can without jeopardizing ultimate recovery, and at the same time keep operating costs at minimal levels. Also, they wish to be free from regulatory constraints whenever there is a reasonable suspicion that ultimate recovery is affected, and/or operating costs are prohibitively increased, and/or first profits are unnecessarily delayed. Hence the relevancy of the question: Do curtailments of various types influence waterflood recovery efficiency?

This paper considers only the question of the rate sensitivity of the waterflood recovery process, but circumvents the related problems of specifying optimum operating rates and plans. The proponents and opponents of curtailments, in general, seem to assume that recovery is influenced by rate; but, apparently, this basic premise still has to be firmly established.

For simplicity, two idealized categories of curtailments might be considered. Holding all other factors constant, a comparison could be made (1) between a low constant rate flood and a high constant rate flood of a linear system, or (2) between a constant (intermediate) rate flood of a linear system, and an interrupted flood of the same system where the rate changes stepwise and cyclically between zero and the given constant rate at some appropiate frequency. Such problems are considered qualitatively in this paper.

For the purpose of these analyses, it is useful to review the opinions of former workers and to develop certain basic theory that heretofore has not been considered.

REVIEW OF PAST WORK

Current opinion and a cross section of opposing viewpoints on the importance of rate in waterflooding is given in papers by Jordan et al. (1957) and Buckwalter et al. (1958). The term "rate, " of course, is ambiguous because it varies continually with time and position during the unsteady-state stage, and with position from the injection to the producing wells during the steady-state stage of waterflooding. Rate also may refer to flood-front advance rate, water injection rate, oil production rate, or total fluid production rate.

Jordan et al. (1957) give an extensive review of the literature, citing more than fifty references, and interpret various field and laboratory observations, concluding that:

(1) "High rates of injection with capacity production are not necessary to obtain maximum recovery."
(2) "Water floods can be curtailed without loss of oil recovery."
(3) "In natural reservoirs, which usually comprise heterogeneous formations, reduction in the rate of water advance may enhance recovery as a result of capillary forces to produce more uniform flooding."

Buckwalter et al. (1958) review essentially equivalent material, but conclude:

(1) "Effect of restricted rates on oil recovery by water flooding is a damaging effect. Oil recovery is reduced by restrictions of rates."
(2) "If the water-flood operator does not receive an early return on his investment dollars, he cannot afford to obtain his oil at necessarily high lifting costs over an extended period of years, and he will, therefore, leave oil in the reservoir due to prematurely reaching the economic limit."
(3) "It is not necessary to prove a physical loss of oil by low production rates in water floods (although this can be proven, too) in order to show wasted oil at low rates of production. Any oil not obtained before the economic limit of a water flood is lost oil and not subsequently recoverable at a profit."
(4) "Operators are absolutely correct in their contention that interrupted, prorated, or low-pressure floods produce less ultimate oil than uninterrupted high-pressure floods. This relationship between rate of production and ultimate recovery can be clearly shown by analyses of flood results."

Surely such divergence of opinion must result from some degree of misinformation, misunderstanding, and mistaken interpretation. As field and laboratory data are characteristically inconclusive, and waterflooding theory is complex and difficult to formulate and understand, errors in evaluation are expected, perhaps inevitable. Also, vested interests make bias likely, even when it is not intended. Large operators, favored with a high ratio of primary to secondary recovery production, do not have the motivations and incentives of the smaller operators who depend on a quick capture of waterflood oil. That both sides righteously claim to be fostering conservation is an enigma that cannot be considered here, however.

It is relevant to examine these paraphrased assertions of Jordan et al.:

(1) Comparable mechanisms of displacement characterize both primary water drives and secondary waterflooding.

(This view is both tenable and useful, as will be seen if account is taken of the initial saturation conditions.)

(2) Waterflooding of homogeneous sands is not rate sensitive. The rate sensitivity, when observed, is a consequence of the heterogeneous character of reservoir sands as found in nature; moreover, restricted recovery rates may enhance recovery efficiency in such sands.

(On the surface, these postulates appear to be too simple and all inclusive to be valid; hence, they are analyzed in a later portion of this paper.)

(3) The ultimate recovery increases without limit as the flood-front rate decreases.

(This is an endorsement of the view set forth by Fancher (1950) and must be closely scrutinized.)

(4) Mobility ratio between displacing and displaced phases controls waterflood recovery in a way that is not rate sensitive.

(This is not consistent with the indications of theory, as will be shown.)

(5) A plot of cumulative oil production versus cumulative water injection is more appropriate than decline curve analysis in predicting recovery efficiency.

(Interestingly enough, Jordan et al. are willing to use decline curve analysis [cf. their figs. 11 and 12] to establish a favored point, and they fail to recognize the masking of detail that invariably results from the use of cumulative plots. In fact, any extrapolation based on an empirical correlation is of doubtful validity because it excludes explicit reference to causative factors. It is likewise useless to look for the effect of a single variable in the cumulative plot of field data unless all other variable factors have been held constant. Such control is seldom possible.)

(6) During the initial stages of displacement, capillary effects in the individual pores at the flood-front boundary control what happens in a way that cannot be altered by a variation in the injection pressure.

(The half truth of this proposition is appealing; but, because of contact angle hysteresis, it cannot be accepted as valid.)

(7) Residual oil initially trapped as microscopic globules cannot thereafter be removed by reasonable velocities of contiguous flowing water.

(This is likely, and is important in emphasizing that microscopic displacement is complete as soon as the filamentary paths of communication are broken. Presumably, it occurs early in the primary stage of flood front advance, suggesting that the so-called subordinate phase production is related to the time-dependence of the sweep efficiency [and/or to the unequal rates of displacement in layered zones of different permeability].)

(8) Relative permeabilities, and hence, waterflood recoveries, are rate independent.

(This depends upon the unverified assumption that laboratory-determined [steady-state] values of relative permeability can be used directly in the Buckley and Leverett (1942) frontal-drive equation. In dispute here is the idea that qualitative conformance is proof of quantitative equivalence.)

(9) The existence of capillary and gravity forces results in recovery's being rate sensitive only when the rates are outside the range normally encountered in field practice.

(This conclusion rests on laboratory findings cited by Jordan et al. Their admission of the existence of a dependence is considered more relevant than their reported experimental results, as the latter must still be verified by other workers with other standards regarding the adequacy of scaling. The cited data about the effect of a free gas saturation may be similarly regarded.)

(10) In layered strata of different permeability that lack vertical communication except at well bores, waterflood recovery is rate independent.

(This position is inconsistent with the pore-doublet theory of Rose and Witherspoon (1956) and Rose and Cleary (1958), which implies that rates must be considered [see below].)

(11) When strata of differing permeability have vertical communication throughout (that is, between well bores), high flooding rates minimize the beneficial effect of the high permeability zone acting as a reservoir from which water can imbibe (by capillarity) to increase recovery from the low permeability zone.

(This observation is consistent with pore-doublet theory, in a way related to the fact that some experiments [Jordan et al.; Richardson and Perkins, 1957] were done under conditions of an unfavorable mobility ratio. In both experiments the viscosity of the displaced oil was greater than the viscosity of the imbibing water [ratios of 0.72 and 0.49, respectively]. Pore-doublet theory, as presently developed, does not pertain to the situation where viscous fingering can occur, but it does deal with mobility ratio effects. The experimental results are defective in both cited cases because the more permeable sand strata lay above the less permeable strata. This chance condition favors a more pronounced gravity segregation during the low rate of flooding [from the upper permeable strata downward] than during the high rate, an effect that would tend to increase the initial recovery efficiency. Inverting the layer would avoid this special effect, and would probably show lower increased recovery efficiency at low rates, according to pore-doublet theory. In any case, no general conclusion of universal applicability can be drawn from these cited results.)

(12) Low rates are more favorable to increased recovery from a tight lens surrounded by permeable sand than are high rates.

(This holds principally for capillary imbibition [zero injection pressure] in a direction opposing the gravity force field, a fact that has been overlooked since the earlier work of Buckley and Leverett (1942). A lessened rate effect should be expected in practice, in accordance with pore-doublet theory, notwithstanding the indications of improperly scaled laboratory experiments.)

In questioning the above points, the foregoing analysis is not intended to refute the basic contention of Jordan et al. that some previous workers have misunderstood the importance of rate on waterflood recovery. Neither is it intended to be critical of a summary of opinion which is by far the most comprehensive and authoritative yet to appear. The questions have been raised to introduce the theoretical views that follow. Indeed, similar questions arise in connection with the points discussed by Buckwalter et al. (1958) which may be paraphrased as follows:

(1) The "excess allowable" represents less than one percent of the total production in Texas, and less than two percent in Oklahoma.

(This observation is scarcely a major point and its presentation as such is begging the question.)

(2) An operator cannot fail to consider the economic side of the picture.

(As one man's medicine is another man's poison, this appears to depart from the central question about the effect of rate on recovery efficiency. Once this effect is determined, it is a simple matter to formulate programs consistent with good economics as well as proper conservation.)

(3) Arbitrary selection of examined field data and arbitrary methods of data representation have been used.

(Both Buckwalter and Jordan and their associates have made charges and counter-charges regarding arbitrary methods used. Their criticism is both timely and valid and refers also to the inconclusiveness of field data, and to a need for an improved theoretical insight pertaining to the mechanism of the recovery process.)

(4) To be successful, waterflooding must be a continuous, uninterrupted operation. Any interruption of the flood will result in immediate and ultimate loss of oil, whether the shut-down is caused by mechanical difficulties or arbitrary regulation.

(Such statements are too general to have anything more than accidental and/or occasional application in particular cases.)

(5) Under conditions of low injection pressure only a portion of the reservoir will be flooded with water, and as pressure is increased this portion increases.

(While it is true that the gravity sweep efficiency will be highest at high injection rates, a high displacement efficiency is favored by low injection rates; moreover, in homogeneous sands, the horizontal sweep efficiency will be independent of rate.)

(6) Low permeability pay zones are bypassed (with a loss in ultimate oil recovery) when the injection rates and the pressures are low.

(Such a consequence can be expected only if the low permeability zones are preferentially oil wetted.)

(7) It is questionable whether or not laboratory data obtained to date are good enough in quality to measure accurately the degree of waterflooding rate-sensitivity expected in field operations.

(Buckwalter et al. raise the point that core wettability may differ from that which exists in the prototype reservoir, and these authors suggest that there are scaling errors in most laboratory experiments. It should not be forgotten, however, that even when laboratory data fail to show quantitative effects, they nevertheless may show the direction of the effects in a qualitative [order-of-magnitude] way.)

An attempt is made here to develop by theory the connection between what actually happens in the field and what is observed and/or assumed to happen during the course of laboratory and field experiments. In doing this the importance of capillary effects will be emphasized.

RECOVERY PROCESS

General Features

Before discussing the effect of curtailments on waterflooding, both macroscopic and microscopic aspects of petroleum recovery must be visualized and described. The recognized conditions for the recovery of petroleum include the following:

(1) The underground sedimentary rock must be able to contain hydrocarbon fluids. That is, its porosity must have been preserved since its formation so that ancient driving forces such as compaction and capillarity could have forced into it oil from a nearby hydrocarbon source bed.

(2) The underground petroleum reservoir must also be a conductor for the flow and transfer of fluids. In general, this is a path requirement (Rose, 1954); that is, whenever the relative permeability to the oil is greater than zero, a continuous path exists in the porous rock through which oil can flow. In other words, the tortuosity is not infinite, the oil saturation is finite and greater than some minimum value.

(3) Driving forces must exist that have negative gradients toward the well-bore sinks, so that the oil in the reservoir will flow through the available paths. Categories of driving forces have been listed (Rose, 1954) as including body forces

(such as gravity), capillary forces, and the forces per unit area arising from pressure differences along flow paths (for example, initial reservoir pressure minus well-bore pressure). Mass transfer, possibly by diffusion, also may occur, but these effects are ignored in the following analysis. In any case, as energy is always spent in the transport of oil through porous rock, potential energy must "pre-exist" in the reservoir (as the potential energy of position, or of a compressed fluid and/or rock, or of a hydrophilic but oil-saturated pore space, or similar conditions); or energy must be supplied (as by the injection of a displacing fluid). The energy gradient, thus, is the driving force, and the energy gradient per unit volume is the potential, V, which appears as the driving force in Darcy's Law as

$$\bar{v} = -\frac{k}{\mu}\,\text{grad}\ V \tag{1}$$

where $\bar{v}$ is the local vector fluid velocity, k is the permeability (conductivity) to the flow of a particular fluid, and μ is the viscosity of that fluid. Explicitly, V may be thought of as being determined by the algebraic summation of the hydrodynamic pressure, the hydrostatic pressure, and the capillary pressure.

(4) Ignoring mass transfer, the recovered oil withdrawn through well-bore sinks must be replaced by some other phase back in the reservoir where the oil originated. This feature is apparent in the case of fluid injection, pressure-maintenance operations. Less obvious examples of this replacement concept include:

(a) Pore-volume shrinkage resulting from the decrease in pore fluid pressure during recovery, thus permitting subsidence of overlying sediments and/or the expansion of compressible mineral elements making up the grain volume. Note that bulk-volume shrinkage occurs in the first case, but not necessarily in the second.

(b) Replacement of the vacated space by gas which comes out of solution as pressure is decreased.

(c) Replacement of vacated space by the expansion of the connate reservoir fluids. Edge and bottom waters expand (and/or flow) into the vacated space during natural water drives. Similarly, an expanding gas cap replaces oil during gravity drainage. Removed oil also can be replaced by the expanded volume of contiguous oil, at least during the initial stages of the recovery process.

Several of the replacement mechanisms described above can have a sequential role (in time as well as space) during oil recovery.

The foregoing analysis summarizes the general features of the recovery process in which the effects of rate of recovery and the consequent efficiency of the process are determined by a host of pre-existing and/or prearranged initial and boundary conditions. The complete description of the recovery process, however, must also consider certain microscopic details, especially the heretofore neglected subject of surface chemistry capillarity. The details of the pore structure must be known before the capacity of the reservoir to contain and transmit fluids can be calculated. The surface chemistry interactions at local fluid-fluid and fluid-solid interfaces also must be known before the displacement and replacement mechanisms can be understood.

Let us consider a random fluid particle, for example, an incremental volume element, contained somewhere in the free volume of the porous continuum. Ignoring the kinetic motion of the fluid particle molecular elements, the particle may be considered stationary unless some net force is acting. Indeed, if it lies strongly sorbed to some pore-wall (solid) surface, no subsequent net force may be encountered that can cause it to move. If force gradients exist, however, the direction of the resultant motion will be determined vectorially, as will the rate of the resultant motion, upon specification of the prevailing resistances.

For example, the fluid particle may receive the impulse to move forward from a contiguous preceding particle in the same flow tube, the chain of this movement being the consequence of the loose piston action of some far-removed displacement disturbance. Similarly, because of cohesion, the fluid particle may be drawn forward through a chain that has its origin in the adhesional wetting action of some far-distant fluid-solid interfacial boundary. Finally, motion may be up or down because of the superimposed gravity effect, and/or because of the sinuosity of the pore-wall boundaries.

In all these movements, we are concerned only with laminar flow, which means that the rate effects will be influenced by the proximity of the fluid particle to contiguous solid surfaces through viscous drag resistance. That is, the microscopic directional changes as the fluid moves along the tortuous, involuting and convoluting pore path is ignored as far as any supposed consequent dependence on the inertia terms applicable to turbulent flow is concerned.

The integrated effect of all such motions results in a transfer by flow of particular fluids through porous media. Mass transfer effects may be superimposed but are ignored here. To express the relevant equations of motion for the most general case, Muskat (1949) combines Equation 1 with the appropiate continuity statement to obtain

$$\nabla \left[\frac{\rho k}{\mu} \nabla V\right] = f \frac{\partial \rho}{\partial t} \qquad (2)$$

where ρ = fluid density, t = time, and f = porosity.

When we concern ourselves with multiphase flow, the following set of equations apply (Muskat, 1949):

$$\bar{v}_G = -\frac{k_G}{\mu_G} \nabla (p_G - \rho_G gh)$$

$$\bar{v}_O = -\frac{k_O}{\mu_O} \nabla (p_O - \rho_O gh) \qquad (3)$$

$$\bar{v}_W = -\frac{k_W}{\mu_W} \nabla (p_W - \rho_W gh)$$

$$S_W + S_O + S_G = 1 \qquad (4)$$

$$p_O - p_W = P_{c\,O/W}$$

$$p_G - p_O = P_{c\,G/O} \qquad (5)$$

$$p_G - p_W = P_{c\,G/W}$$

where S is the fluid saturations per unit pore volume; p is pressure; P_c is the so-called capillary pressure; and the subscripts G, O, W refer, respectively, to the gas, oil, and water phases.

From equations (3), (4), and (5) the relevant equations of motion in multiphase saturated media can be derived, as given by Muskat (1949).

Surface Energy Approach

Any interface between two immiscible or otherwise separated phases possesses a free energy that has a constant value per unit area at equilibrium when temperature is constant. The existence of this surface energy is revealed, for example, by the work which must be done to crush solids, because new surfaces are being created. The heats of wetting and adsorption also reflect a change of surface energy as one fluid-solid interface is replaced by another. Finally, the observation that liquid-vapor interfaces assume a shape of minimum area compatible with the constraints of the system is expressive of the tendency of the system to reach a state of minimum free surface energy. Thus, the free surface energy may be thought of originating because molecules near the surface have a higher potential energy than molecules in the interior of the liquid (or solid) phase, due to the evident unbalance of molecular forces at surface boundaries. Values of surface energy per unit area are expressed by the familiar interfacial tensions, where this designation denotes the dimensional equivalence between ergs for cm^2 and dynes per cm.

As far as oil recovery is concerned, most of the early misinformation on the role of surface forces is reviewed in a paper by Fancher (1950).

Petroleum reservoirs are systems in which interfaces between solids (S), water (W), oil (O), and gas (G) may be found in abundance. For example (Rose and Bruce, 1949), the surface area of a sedimentary sandstone is inversely proportional to the average particle size, and may reach values as great as 20, 000 cm^2 per unit bulk volume (for example, 10, 000 cm^2 per gram). This tremendous surface area will be composed of interfaces between solid-water (SW), solid-oil (SO), and solid-gas (SG), according to the formula

$$A_S = A_{SW} + A_{SO} + A_{SG} = \text{constant} \qquad (6)$$

where A is the specific surface area (per unit bulk volume), and A_S is the constant value of the total specific surface area (per unit bulk volume).

In addition to the three possible solid-fluid interfaces, there are three possible fluid-fluid interface types that can exist in the general case, namely the water-oil (WO), the water-gas (WG), and the oil-gas (OG) interfaces. Thus

$$\begin{aligned} A_W &= A_{SW} + A_{WO} + A_{WG} \\ A_O &= A_{SO} + A_{WO} + A_{OG} \\ A_G &= A_{SG} + A_{WG} + A_{OG} \end{aligned} \qquad (7)$$

Table 1. - Classification of Interface Possibilities in Multiphase Saturated Porous Media

See fig.	Code	Position of liquid 1	2	3	Possible double points	Possible triple points	Remarks
1A	A1	W	O	-	SW, WO	-	a
1A	A2	O	W	-	SO, WO	-	b
1A	B1	W	G	-	SW, WG	-	c
1A	B2	G	W	-	SG, WG	-	c
1A	C1	O	G	-	SO, OG	-	d
1A	C2	G	O	-	SG, OG	-	d
1B	D	W	O	-	SW, SO, WO	SWO	e
1B	E	W	G	-	SW, SG, WG	SWG	c
1B	F	O	G	-	SO, SG, OG	SOG	d
1C	G1	W	O	G	SW, WO, OG	-	a
1C	G2	W	G	O	SW, WG, OG	-	b
1C	G3	O	W	G	SO, WO, WG	-	b
1C	G4	O	G	W	SO, WG, OG	-	d
1C	G5	G	W	O	SG, WO, WG	-	d
1C	G6	G	O	W	SG, WO, OG	-	d
1D	H1	W	O	G	SW, WO, WG, OG	WOG	a
1D	H2	O	W	G	SO, WO, WG, OG	WOG	b
1D	H3	G	W	O	SG, WO, WG, OG	WOG	d
1E	J1	W	O	G	SW, SO, WO, WG, OG	SWO, WOG	e
1E	J2	W	G	O	SW, SG, WO, WG, OG	SWG, WOG	e
1E	J3	O	G	W	SO, SG, WO, WG, OG	SOG, WOG	e
1F	K1	W	O	G	SW, SO, SG, WO, OG	SWO, SOG	b
1F	K2	W	G	O	SW, SO, SG, WG, OG	SWG, SOG	b
1F	K3	O	W	G	SW, SO, SG, WO, WG	SWO, SWG	d
1G	L1	W	O	G	SW, SO, SG, WO, WG, OG	SWO, SWG, WOG	d
1G	L2	O	W	G	SW, SO, SG, WO, WG, OG	SWO, SOG, WOG	b
1G	L3	G	W	O	SW, SO, SG, WO, WG, OG	SWG, SOG, WOG	d

S = solid phase.

W = aqueous phase.

O = liquid hydrocarbon phase.

G = gaseous phase.

a = a configuration to be found commonly in water-wet systems.

b = a configuration seldom encountered.

c = a system not relevant to the oil recovery problem.

d = a configuration probably never encountered.

e = a configuration to be found commonly in systems of mixed wettability.

Table 1 and figure 1 show that 27 possible configurations of these six interfacial possibilities* exist, some of which are more likely to be found in petroleum reservoirs (initially and/or later during oil recovery) than others. We must therefore consider the A and D systems where no free gas phase exists, but we may ignore the possible occurrence of C and F systems which assume the existence of no connate water. As oil is absent in the B and E systems, these also may be disregarded in the following discussion of oil recovery. Finally, the A2 system may be disregarded to the extent that completely oil-wet systems are not expected in nature. We shall therefore be interested in only two of the nine possible two-fluid phase systems (the A1 and the D systems).

Table 1 shows that there are 18 possible three-fluid phase systems, but many of these also are of unlikely occurrence. In completely water-wet systems we may expect to find the G1 and G2 configurations, the latter for reasons cited by Muskat (1949), who suggests that the gas phase sometimes changes position with the oil as the oil saturation decreases. We may also expect the H1 and H2 configurations for water-wet and partially oil-wet systems, respectively. Thus, although the G3 configuration is not to be expected, the J1 and J2 configurations might be encountered in mixed-wettability systems. The K configurations are most unlikely in any given small volume element, which does not rule out the

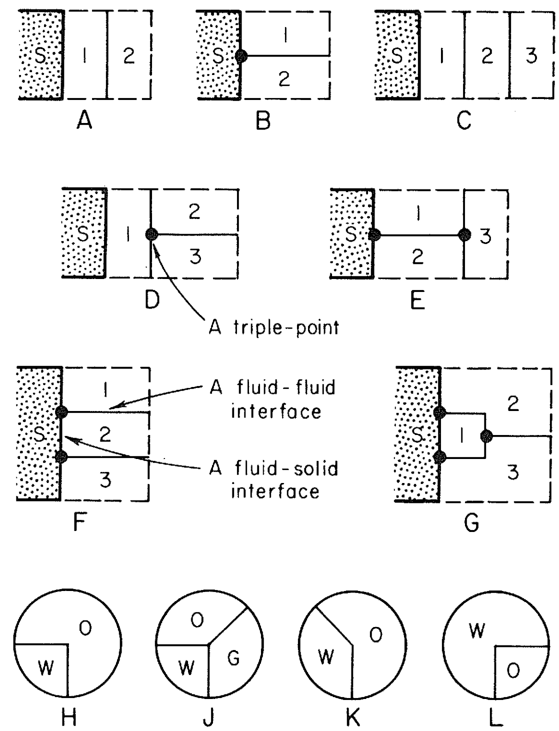

Fig. 1. - Sections A through G show the possible interfacial configurations listed in table 1. Diagrams H through L depict likely saturation changes during depletion of solution gas (H to J), during the fill-up period as water is injected forcing free gas back into solution (J to K), and during waterflooding (K to L). Probable interfacial configurations during this sequence might be: types A1 and D for diagram H representing the virgin reservoir condition; types G1, G2, H1, J1, J2, or K1, for diagram J representing conditions after depletion drive; and types A1 and D for diagrams K and L, representing conditions after fill-up, and after waterflooding, respectively.

* In mixed mineral systems, various solid-solid interfaces also exist, each having finite free surface energy. These, however, can be ignored in discussing oil recovery because the solid-solid interfacial areas do not change with time.

possibility that in a large reservoir we may find water, oil, and gas each lying next to interstitial (solid phase) surfaces in different regions. The L2 configuration, however, might be encountered at the end of a recovery process where a small globule of residual oil is stuck to a pore wall (and elsewhere surrounded by similar volumes of water and gas).

Table 1 also lists the double-point and triple-point possibilities associated with the various interfacial configurations. Double points occur where two immiscible phases have a line, or surface, of contact and triple points occur where three immiscible phases have a point, or line, of contact. As mentioned, there are six double-point possibilities. Of these, the SW, WO, and OG contacts are the most common, the WG contact will be found, and the SO and SG contacts occur less frequently. Of the four possible triple points, all are possible but the SWO contact is most likely.

To illustrate the importance of surface energy as an important feature of the reservoir system, assume that the total area of all types of interfaces (per unit bulk volume) equals 10^4 cm^2, and that the mean value of energy for these interfaces equals 50 ergs per cm^2. Then the free energy content of the system equals about 6×10^{14} ergs per acre-foot (equivalent to about 4.5×10^7 foot-pounds of work per acre-foot of reservoir volume). Only that portion of the free surface energy associated with fluid-fluid interfaces is available to do work in the recovery of oil, as will be shown. Leverett's early results (1941) suggest that the fluid-fluid interfacial area in certain cases may be comparable to the solid-fluid interfacial area; hence, in the present example, if we are considering a 10, 000 acre-foot reservoir, we are dealing with more than 200 billion foot-pounds of energy available to do work due to the existence of fluid-fluid interfaces alone. If this 10, 000 acre-foot reservoir is 5, 000 feet deep, and characterized by a porosity of 30 percent and an oil saturation of 70 percent, the surface energy would be equivalent to about one percent of the foot-pounds of work required to lift the oil (of unit density) to the surface. Actually, as oil reaching the well-bore generally is brought to the surface by the potential energy of an artesian system, the surface energy is to be thought of as utilized only to do the work of overcoming viscous resistance in the movement of the oil through the reservoir pore spaces (Rose, 1957).

The role of surface energy in the movement of oil through petroleum reservoirs can be formulated analytically. From the definition of interfacial tension as the free free energy per unit area, we have

$$F = \gamma_{SW}A_{SW} + \gamma_{SO}A_{SO} + \gamma_{SG}A_{SG} + \gamma_{WO}A_{WO} + \gamma_{WG}A_{WG} + \gamma_{OG}A_{OG} \qquad (8)$$

where F is the free energy per unit bulk volume, A is the surface area per unit bulk volume, and γ is the appropriate interfacial tension.

Applying the Neumann Triangle of Forces (fig. 2), we may obtain the vectorial balance of forces at the SWO, SWG, SOG, and WOG triple points, as

$$\begin{aligned} \gamma_{SO} &= \gamma_{SW} + \gamma_{WO} \cos \theta_{WO} \\ \gamma_{SG} &= \gamma_{SW} + \gamma_{WG} \cos \theta_{WG} \\ \gamma_{SG} &= \gamma_{SO} + \gamma_{OG} \cos \theta_{OG} \\ \gamma_{WG} &= \gamma_{WO} + \gamma_{OG} \cos \theta_{OG} \end{aligned} \qquad (9)$$

where θ is the appropriate contact angle.

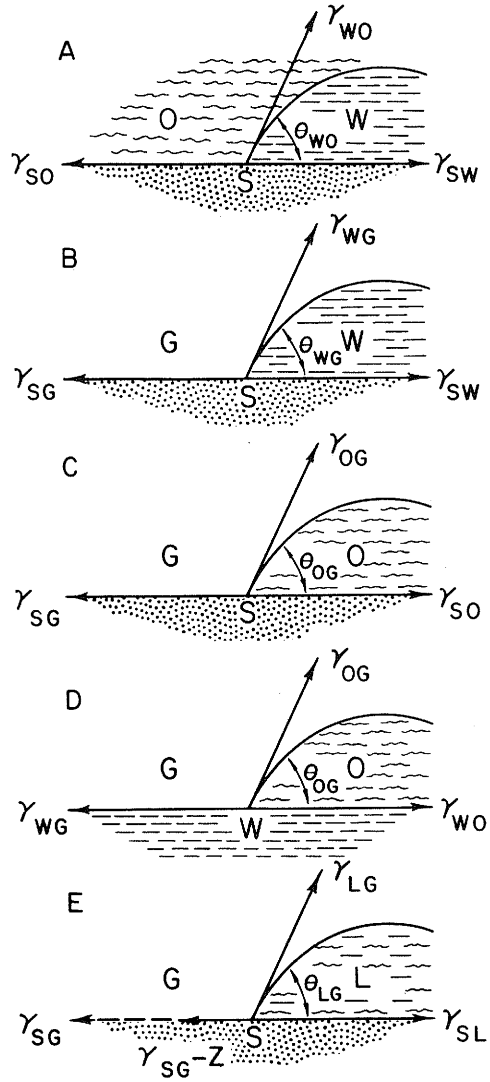

Fig. 2. - Neumann Triangle of Force law illustrated for various interfacial configurations. Diagrams A through D apply to equation 9, and diagram E applies to the situation where the energy of the solid-vapor surface has been decreased by -Z due to adsorption.

Combining equations (8) and (9) gives the general expression for the free surface energy of a reservoir system due to the existence of solid-fluid (SF) and fluid-fluid (FF) interfaces, as

$$F = A_S\gamma_{SW} + A_{SO}\gamma_{WO}\cos\theta_{WO} + A_{SG}\gamma_{WG}\cos\theta_{WG} + A_{WO}\gamma_{WO} + A_{WG}(\gamma_{WO} + \gamma_{OG}\cos\theta_{OG}) + A_{OG}\gamma_{OG} \quad (10)$$

When no free gas phase is present, equation (10) reduces to

$$F = A_S\gamma_{SW} + A_{SO}\gamma_{WO}\cos\theta_{WO} + A_{WO}\gamma_{WO} \quad (11)$$

The differential forms of equations (10) and (11) are useful for the specification of capillary flow in reservoir rock, because of the inviolate tendency of systems in nature to reach states of minimum energy for which an approach path exists, through loss of energy to the surroundings, or through the equilibriation of the free energy content between all points in a closed system.

In the present case where we are considering the free surface energy of fluid-filled porous rock, we may note that the partial of F with respect to distance along a given X-direction, at constant time, determines the flow direction according to

$(\partial F/\partial x)_t < 0$ implies flow in the X-direction.

$(\partial F/\partial x)_t > 0$ implies flow in the minus X-direction.

$(\partial F/\partial x)_t = 0$ implies no net flow in the X-direction. (12a)

Similarly, the partial of F with respect to time, for a given X coordinate position, determines the magnitude of the driving force, according to

$(\partial F/\partial t)_x < 0$ implies that spontaneous capillary flow (an imbibition process) occurs.

$(\partial F/\partial t)_x > 0$ implies that work is being done by external forces (a drainage process) increasing the energy of the system.

$$(\partial F/\partial t)_x = 0 \text{ implies that equilibrium has been reached.} \tag{12b}$$

Accordingly, the differential form of equation (10) may be abbreviated as

$$\partial F_{total} = \gamma\Sigma\partial A = \gamma_{WO}\partial A_{WO} + (\gamma_{WO} + \gamma_{OG} \cos \Theta_{OG})\, \partial A_{WG} + \gamma_{OG}\partial A_{OG} + (\gamma_{WO} \cos \Theta_{WO})\, \partial A_{SO} + (\gamma_{WG} \cos \Theta_{WG})\partial A_{SG} \tag{13}$$

which reduces to the following special cases:

$$\begin{aligned} &(\text{no gas present}) \quad \partial F = \gamma_{WO}\partial A_{WO} + \gamma_{WO} \cos \Theta_{WO}\,\partial A_{SO} \\ &(\text{water-wet systems}) \quad \partial F = \gamma_{WO}\partial A_{WO} \\ &(\text{mixed wettability systems}) \quad \partial F = \gamma_{WO} \cos \Theta_{WO}\,\partial A_{SO} \end{aligned} \tag{14}$$

Note that in these three cases $\partial A_{SO} = -\partial A_{SW}$. Therefore the second of equations (14) is based on the idea that oil never touches the pore wall (hence, A_{SO} is always zero), and the third assumes that A_{WO} has a constant value. The latter assumption is expected when the contact angle is finite; it also represents the condition that holds during capillary rise in uniform bore tubes.

The net force acting in the X direction on a particular fluid-fluid (FF) interface is given by summing the differentials of each term of equation (10) with respect to surface area, and multiplying these energy gradients per unit area times the perimeter, P, in the plane normal to the direction the force is acting. Thus

$$Q = \Sigma \frac{\partial F}{\partial A} \cdot P \tag{15}$$

where Q is the force. To illustrate, water will imbibe into an oil-filled capillary tube with a force equal to $2\pi R\gamma_{WO} \cos \Theta_{WO}$ (as found from the third of equations 14) and, if the direction is vertically upward, this force will be opposed (and ultimately balanced) by a downward gravity force equal to mass times gravity acceleration times height of capillary rise. This leads to the familiar definition of capillary pressure, P_c, which holds for zero contact angle situations in circular capillaries, as

$$P_c = \frac{2\gamma_{WO}}{R}$$

where R is the radius.

Capillary pressure is defined (equation 17) as the pressure difference across curved interfaces separating immiscible fluid phases. The wetting fluid phase is identified as that through which a contact angle, Θ, of less than 90° is measured; conversely, the nonwetting fluid phase is that through which a contact angle greater than 90° is measured. The higher pressure invariably exists on the concave side of the interface, which is generally in the region occupied by the nonwetting phase. Exceptions are illustrated by figure 3, based on the original treatment by Adam (1948), which indicates that the nature of the interfacial curvature will depend on the interspatial geometry of the pore-wall boundaries. Considering only pore spaces having axial symmetry normal to the curved surface of the fluid-fluid interface, and identifying the angle made by the axis and the tangent to the pore wall at the point of contact between the fluid-fluid interface and the solid by ϕ, figure 3 shows:

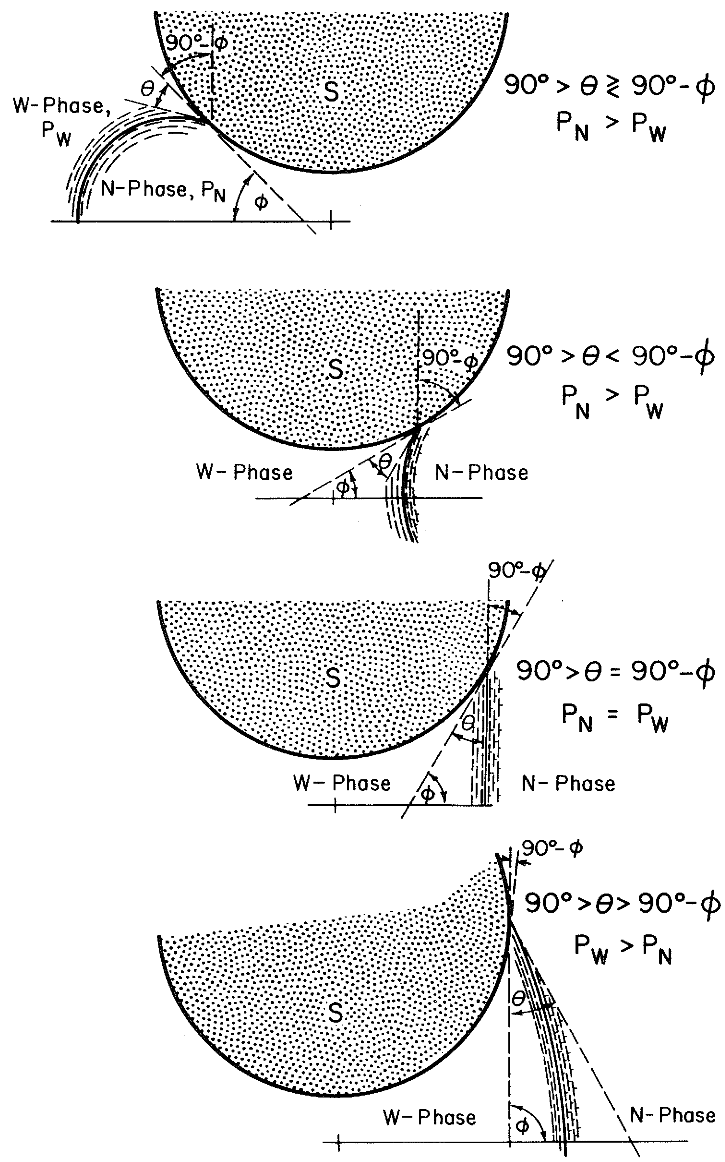

Fig. 3. - The curvature of fluid-fluid interfaces depends upon the geometry of the solid-wall boundaries when the contact angle is fixed and predetermined as in figure 2.

(a) If the pore width immediately on the nonwetting fluid side of the fluid-fluid interface tends to be smaller than the pore width on the wetting fluid side, the interface is always concave towards the nonwetting fluid for both imbibition and drainage processes. Hence, the capillary pressure, taken in the sense as defined by equation (17) below, is always positive, independent of ϕ, and is determined by the magnitude of the appropriate (advancing or receding) contact angle and the principal radii of curvature of the interface.

(b) If the pore width immediately on the nonwetting fluid side of the interface tends to be larger than the pore width on the wetting fluid side, three possibilities can be considered. If the contact angle measured through the wetting phase is less than $90°-\phi$, the capillary pressure is positive as in case (a) above. If the contact angle equals $90°-\phi$, the interface is a plane surface and the capillary pressure is zero. Finally, if the contact angle is greater than $90°-\phi$, the capillary pressure is negative, as the interface is convex towards the nonwetting phase.

Laplace's well known theorem states that surfaces in which the sum of the reciprocals of the two principal radii of curvature equals a constant are surfaces of minimum area. This condition of uniform mean curvature describes fluid-fluid interfacial surfaces existing in nature because minimal surface area means minimal free surface energy. Indeed, it may be shown that the capillary pressure as above defined is equal to the mean curvature of the interface multiplied by the interfacial tension, or

$$P_C = \gamma \left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad (17)$$

where R_1 and R_2 are the principal radii of curvature, defined as the radii of curvature of the two curves formed by the intersection of the surface with mutually perpendicular planes orthogonal to the osculating plane at the given point on the given surface. Moreover, these mutually perpendicular planes are chosen so that the two curves they define on the surface have the maximum and minimum possible curvatures, respectively.

Laplace showed that the surface of a sphere is the only possible minimal surface that is closed; hence, in the absence of other forces such as gravity, spherical raindrops and spherical globules of residual oil are the forms that are found in nature. Other surfaces, such as the surface of a cylinder and the surfaces of revolution of unduloids, catenoids, and nonoids, are also minimal surfaces. These, however, require solid phase boundaries for support. Thus, pendular rings of liquid held by capillarity at points of contact between solid spheres have the catenoid form (Rose, 1958), as does the configuration of a small droplet of liquid bridging the gap between two parallel plates. Similarly, a droplet of liquid on a cylindrical fibre often will assume an unduloid form. Finally, if the solid phase boundary is an undefined three-dimensional curve in space, an infinity of minimal surfaces can be conceived, as illustrated by the variety in form assumed by soap films. Some of these forms are illustrated in figure 4.

In general, fluid-fluid interfacial shapes found in petroleum reservoirs are neither simple nor symmetrical. An exception to this rule is found only in the case of pendular saturation conditions in packings of (nearly) spherical particles where there is no continuity between contiguous pendular rings of liquid. Indeed, the specification of interfacial shape in the funicular regime of fluid-fluid saturation is a hopeless task, as Muskat (1949) has noted. All that can be said is that, at equilibrium, equation (17) holds, although not necessarily as a single-valued

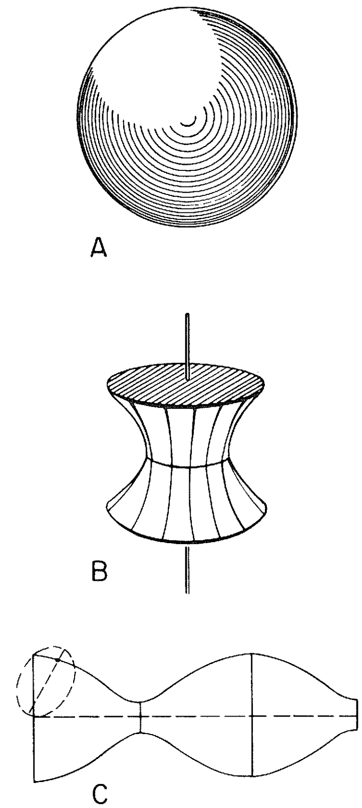

Fig. 4. - Certain minimal surfaces of revolution illustrating the configuration of fluid-fluid interfaces for simple boundary conditions. Diagram A depicts a sphere in which the principal radii of curvature, R_1 and R_2, equal the radius of the sphere. Diagram B shows the catenoid form assumed by a liquid drop held between two parallel plates. Diagram C shows the unduloid form that is the surface of revolution around the axis from which a curve is generated that is the locus of the focus of an ellipse rolling along the axis (such a form is sometimes seen when water drips slowly from a faucet).

function. Well known hysteresis in the saturation versus capillary-pressure relationship establishes that the approach path determines the radii of curvature of the interface, as well as the solid phase boundary conditions, for particular saturation values (fig. 6B). Thus, at particular saturation values, the capillary pressure value is less for the imbibition process than for the drainage process, implying that the energy of the system is less in the former case than in the latter. This illustrates that, in general, more than one minimal surface may rest on given solid phase boundaries if an energy barrier is interposed so that the maximum energy state can persist as a metastable equilibrium point without spontaneous transposition to the lower minimum energy state.

Inasmuch as pressure has the dimensions of an energy gradient per unit area, we may write from the definition of equation (8)

$$P_c = \frac{\partial F_{FF}}{\partial A_{FF}} \cdot \frac{p}{a} = \gamma_{FF} \cdot \frac{p}{a} = \gamma_{FF}\left[\frac{A_{FF} + A_{SF}}{f\,S}\right] \qquad (18)$$

where SF and FF refer to solid-fluid and fluid-fluid interfaces, respectively; p is a planar perimeter closing around the wetting fluid phase; a is the area enclosed by this perimeter curve; f is the porosity; and S is the fractional saturation (per unit pore volume) of the wetting phase. The last segment of equation (18) depends on the frequently used assumption that, in porous media, the p/a ratio can be replaced by a mean hydraulic radius concept, which in explicit form is simply the ratio of the total surface area bounding a tubular form to the volume of that form.

In the limit where S is unity, equation (18) reduces to a form previously given by Rose and Bruce (1949) as

$$\lim_{S \to 1} A_{SF} = A_S = \frac{\lim_{S \to 1} P_c}{\gamma} \cdot f \qquad (19)$$

which was obtained by equating Leverett's J-function (Leverett, 1941) with the Kozeny-Carman equation for flow. At values of S less than unity, equation (18) reduces to a definition of the fluid-fluid surface area per unit bulk volume, as

$$A_{FF} = [\frac{P_c fS}{\gamma_{FF}}] - A_{SF} \tag{20}$$

Note that equation (20) shows A_{FF} going from zero, through a maximum, and back to zero, as S goes from unity to zero, in accordance with expectations (Scott and Rose, 1953). Previously, Leverett (1941) had derived a dimensionally equivalent form

$$A_{FF} = -\frac{f}{\gamma}\int P_c dS \tag{21}$$

Equation (21), however, is defective in the limit that suggests that the fluid-fluid interfacial area is extremely large when S is zero.

Finally, it will sometimes be of interest to combine equation (10) or (11) with equation (13) or (14), to establish the relationship between energy changes and surface area changes. Such a procedure, through integration, invariably will show that the tendency for spontaneous capillary flow is measured by the magnitude of the decrease in free energy of the system brought about as the result of the decrease of certain interfacial surfaces (and perhaps also the increase of others).

Figure 5 illustrates schematically the surface area functions of equation (14) so that some qualitative estimate may be made about the direction of capillary forces during waterflooding. First, it should be noted that zero slopes at finite values have been indicated for the A_{WO} and the A_{SO} curves when the water saturation approaches unity, expressing the idea that the residual oil globules ultimately become discontinuous. Hence, by equation (14) we see that dF/dS is zero, and capillary flow cannot occur. For all other saturation values, figure 5 (in conformance with expectations) shows that dF/dS is negative, since it may be imagined that the A_{SO} term increases with decreasing S much more rapidly than the A_{WO} term decreases. Only in the case where the contact angle, θ_{OW}, approaches 90°, do we expect that dF/dS becomes greater than zero so that capillary imbibition ceases. However, figure 5 shows that if the initial water saturation is high enough, dA_{WO}/dS is already negative and spontaneous capillary imbibition will occur notwithstanding the high value of contact angle.

We may therefore conclude that waterflooding will occur in most cases via capillary imbibition, even in the

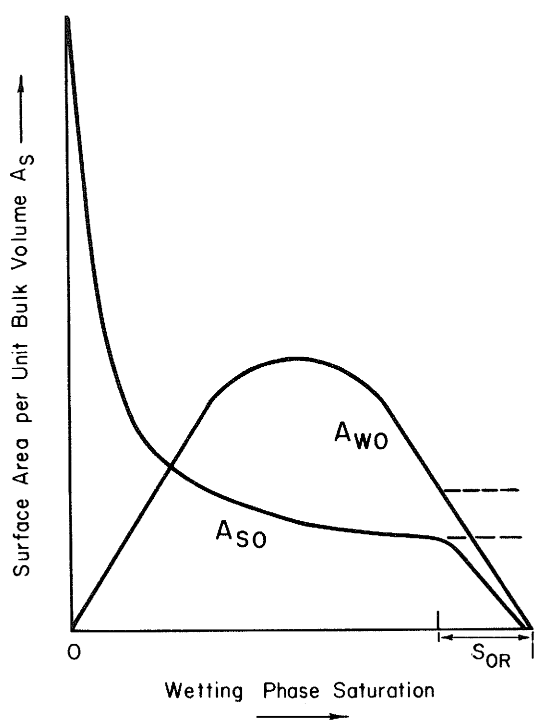

Fig. 5. - A schematic representation of the surface area functions, A_{SO} and A_{WO}, applicable to systems of low contact angle.

absence of other driving forces, as long as there is a source of water and as long as the relative permeability to oil ahead of the flood front is finite. Also, residual oil left behind the flood front as unconnected globules will not be moved by capillary forces, or, indeed, by any other commonly encountered force, as Muskat (1949) and Jordan et al. (1957) have stated.

Figure 5 is intended to represent mixed wettability systems that might be encountered in waterflooding. The A_{SO} term is shown to increase as the oil saturation increases, as would be the case when the contact angle is greater than zero. Although there is no apparent way to evaluate the A_{SO} function for this case, the A_{WO} term of figure 5 might be obtained directly from equation (20).

Examination of the completely water-wet system (θ equals zero) by applying the second of equations (14) and noting that A_{WO} will always be decreasing with increasing values of S affirms again that spontaneous capillary imbibition will occur in the direction of replacing and displacing oil. In this case, it is seen that A_{SW} equals A_S, and A_{SO} equals zero, for all values of S.

The foregoing discussion of surface energy is important in resolving the rate-sensitivity problem because, as can be demonstrated, surface energy gives rise to driving (and resisting) forces that can play a dominant role in the movement of oil through petroleum reservoirs. As noted, the distribution of immiscible fluids (water, oil, gas) in porous rock creates fluid-fluid and fluid-solid surfaces of great magnitude, so that the low energy per unit area (generally less than 100 ergs per cm^2) in fact represents a large total energy content of the system.

This energy content may be illustrated as follows. From equation (15) we note that the force acting on a fluid particle is simply given by the energy gradient in the direction the force acts. Thus, we may write

$$\text{capillary force acting on spherical globule} = \frac{2\gamma_{WO}\cos\theta}{R}\,\frac{dv}{dx} \tag{22}$$

where R is the sphere radius, and v is the sphere volume. The change in volume with distance, dv/dx, is simply the slope of the capillary pressure curve (fig. 6B) and its value may vary from zero to infinity. If an average value of unity is assumed, equation (22) reduces to

$$Q_S = \frac{2\gamma_{WO}\cos\theta}{R} \tag{23}$$

where Q_S is the surface energy (capillary) force.

The gravity force acting on the spherical globule is simply

$$Q_G = \frac{4}{3}\pi R^3 \Delta\rho_{O/W} g \cong 4000\ R^3 \Delta\rho_{O/W} \tag{24}$$

and the hydrodynamic force is

$$Q_H = \frac{4}{3}\pi R^3 \frac{dp}{dx} \cong 4R^3\ \frac{dp}{dx} \tag{25}$$

if the compressibility of the fluid element can be neglected. In equation (25), the pressure gradient term, dp/dx, will have practical limits between zero and 500 dynes per cm^3, whereas the density difference term of equation (24) will have practical limits between zero and 0.2 grams per cm^3.

Letting the fluid particle (globule) radius be 0.1 cm., then Q_S will be 500 dynes if the interfacial tension is taken as 25 ergs per cm^2. This force is to be

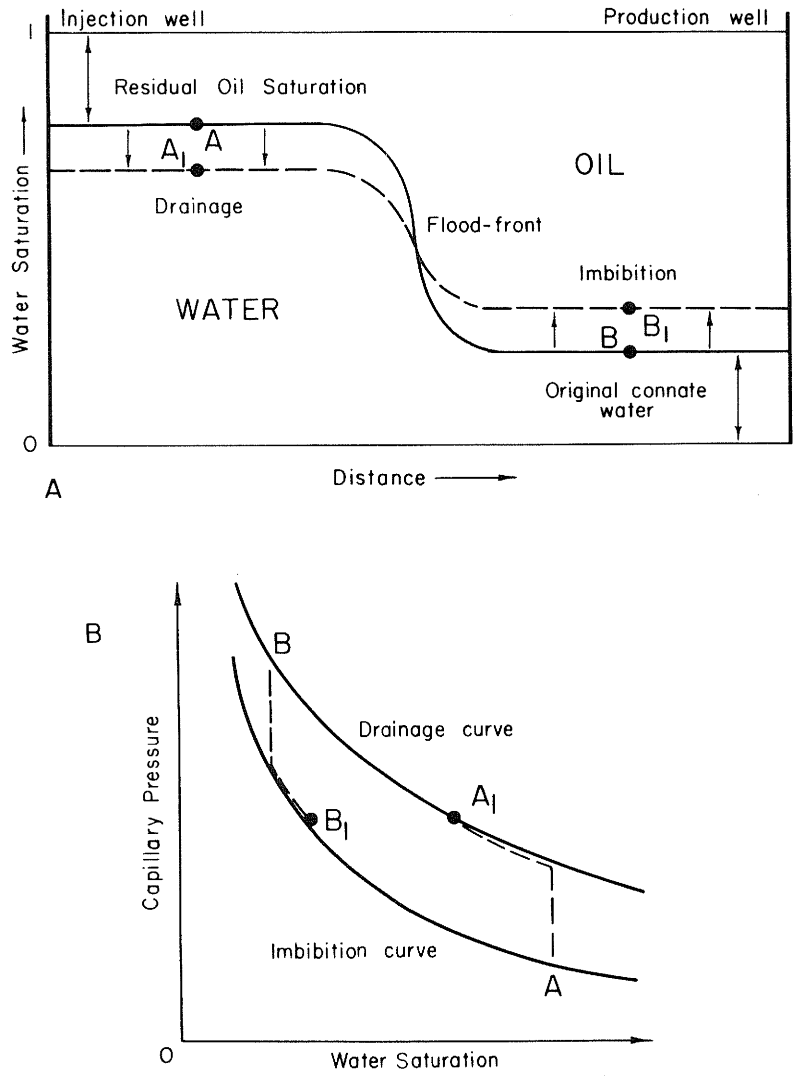

Fig. 6. - A schematic illustration of one effect of interrupted production. In diagram A, the solid curve shows the water saturation profile, with high values at the injection well, intermediate values at the flood front, and low values at the producing well. The dashed curve shows the profile obtained during the shut-in period, due to capillary effects, with drainage occurring behind the flood front and imbibition occurring ahead. The mechanism of saturation changes, from A to A_1 and from B to B_1, is illustrated by reference to the capillary pressure curves of diagram B.

compared with Q_G equals 0.4 dynes (assuming a density difference of 0.1 grams per cm^3) and with Q_H equals 0.04 dynes (assuming a pressure gradient of 10 dynes per cm^3).

In practical cases the density difference might be closer to zero, which would lessen the comparative importance of Q_G. Similarly, the pressure gradient (for example, near injection and producing wells) might be greater than the value that has been assumed above. None the less, it is clear that the resultant force acting in the vicinity of the flood front will always be greatly influenced by the capillary force unless the contact angle approaches or exceeds 90° (oil-wet system), or unless the interfacial tension is negligibly small.

In the recovery process, then, there are three principal energies (disregarding chemical energy, which can cause mass transfer) that can give rise to fluid movement - (1) surface energy (measured by the works of adhesion and cohesion due to the potential energy of molecules located near interfaces in an unbalanced force field); (2) potential energy of position (gravity); and (3) pressure energy measured by the product of pressure multiplied by the volume of an injected fluid.

In nature, closed systems tend towards a state of uniform energy distribution, and unbounded systems will lose energy to the surrounding environment which is at a lower energy state. Such energy transfer presumes the existence of a transfer path.

This is precisely what happens in the petroleum reservoir system during geologic time and during recovery. First, the system will tend towards a state of minimum free surface energy, which means that the immiscible fluids will become distributed in some special fashion, controlled in part by time and in part by the existence of transfer paths. Second, when there is a density difference between the immiscible fluid phases, buoyancy forces (gravity) will force the system toward a state of minimum potential energy. Last, when there is an unbalance in pressure energy (due, for example, to the removal of fluid at one point and/or the injection of fluid at another) this energy will be converted to kinetic energy of motion (and to heat through viscous resistance) until the reservoir attains an even pressure energy distribution.

The gradients of these three energies may be thought of as driving forces, and they have counterparts that may be expressed dimensionally as pressures (or pressure gradients). The relation of the energies and pressures may be stated thus:

1. The existence of free surface energy gives rise to capillary forces and the so-called capillary pressure.
2. The potential energy of position gives rise to gravity forces and to the hydrostatic pressure.
3. The pressure energy gives rise to hydrodynamic forces and to the pressure gradients that exist in a continuous fluid when the hydrostatic pressure differences are subtracted.

From this analysis it therefore may be anticipated that any oil recovery process is inherently rate-sensitive, including the waterflooding variant that will be discussed here. This sensitivity exists because external control of rate does not influence the gravity force field, variably changes the capillary force field, and markedly changes the hydrodynamic force field. The resultant force acting on fluid particles, and causing displacement and transfer, therefore depends on rate both with respect to magnitude and direction.

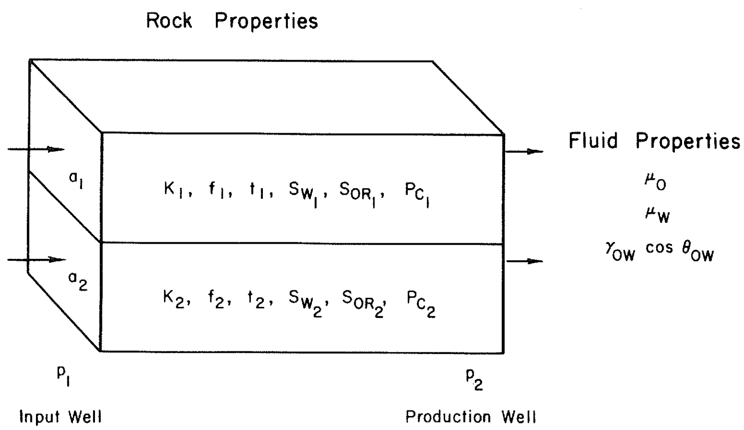

Fig. 7. - Schematic view of a two-layered linear flow system.

Pore Doublet Analogy

Rose and Witherspoon (1956), and later Rose and Cleary (1958) attempted to analyze the implications of the Moore and Slobod (1956) so-called VISCAP theory. Not generally understood is the idea that pore-doublet analysis (and pore N-let analysis) can be applied directly to a consideration of one-dimensional flow and displacement in layered sands where vertical intercommunication (permeability) is negligible.

Given two-layered sands of equal width but different cross-section areas, a_1 and a_2, we shall assume for the general case that the permeabilities, k, the porosities, f, the connate water saturations, S_w, and the residual oil saturations after displacement, S_{OR} all differ in the two layers. Figure 7 shows the geometry of the linear flow system that is being considered.

If the recovery factor, G, is defined as the volume of oil produced from the system at breakthrough and divided by the total initial oil content, the analytic methods of Rose and Witherspoon lead directly to

$$G = \frac{M[a_1 f_1 (1-S_{w_1}-S_{OR_1})] + [a_2 f_2 (1-S_{w_2}-S_{OR_2})]}{[a_1 f_1 (1-S_{w_1})] + [a_2 f_2 (1-S_{w_2})]} \qquad (26)$$

In equation (26), the parameter M is a measure of the distance of flood-front travel in the less permeable layer at the time of breakthrough of the flood front in the more permeable layer. When the displacing water has the same viscosity as the displaced oil, M will be given by

$$M = \left[\frac{\bar{R}^2(\bar{F} + 1)}{(\bar{F} + \bar{R})}\right] \qquad (27)$$

In the more general case of unequal viscosities, M is given by

$$M = \frac{\left[1 + \bar{R}^2\left(\frac{\bar{F} + 1}{\bar{F} + \bar{R}}\right)\ (\bar{V}^2 - 1)\right]^{\frac{1}{2}} - 1}{(\bar{V} - 1)} \qquad (28)$$

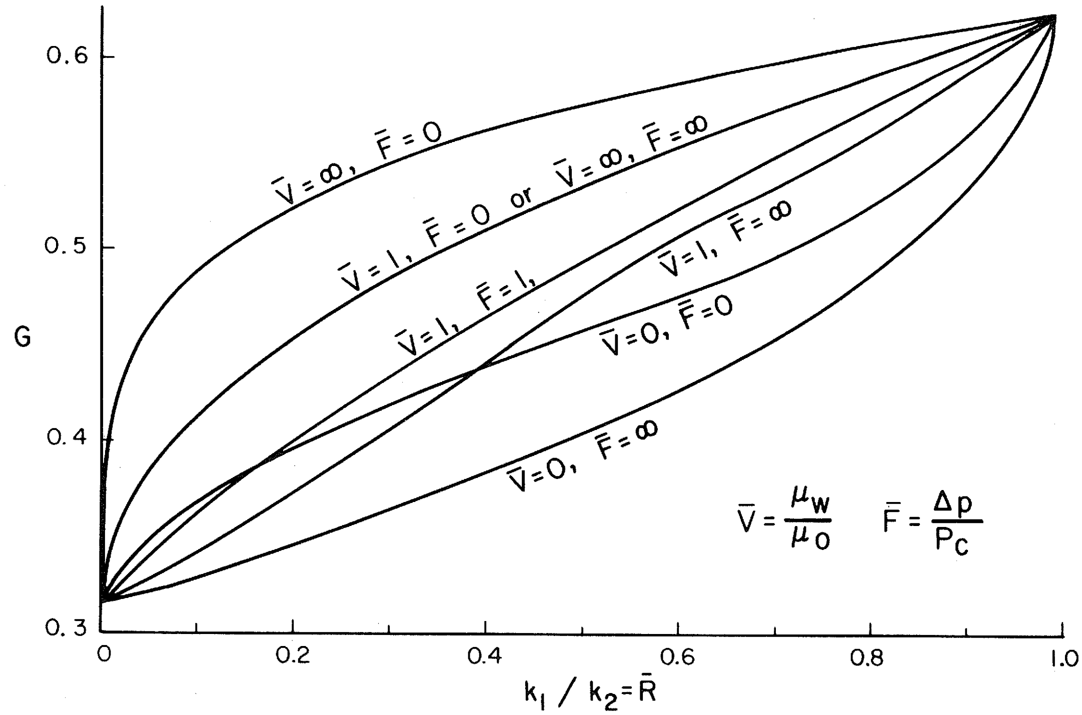

Fig. 8. - Recovery factor, G, versus the permeability ratio, k_1/k_2, for the two-layered flow system of figure 7, where equal porosities and cross-sectional areas have been assumed, where the connate water saturation in each layer is 0.2, and where the residual oil saturation in each layer is 0.3.

In equations (27) and (28), $\bar{V}$ is the ratio of water to oil viscosity; $\bar{F}$ is the ratio of the imposed pressure drop (the difference between pressure at the injection well and at the producing well) to the capillary pressure existing at the flood front in the less permeable sand layer; and $\bar{R}$ is given by

$$\bar{R} = \left[\frac{K_1 f_2 t_2}{K_2 f_1 t_1}\right]^{\frac{1}{2}} \left[\frac{(1-f_1)\ (f_2)}{(1-f_2)\ (f_1)}\right] \cong \left[\frac{K_1}{K_2}\right]^{\frac{1}{2}} \qquad (29)$$

as derived from Kozeny-Carman theory (Scheidegger, 1957). Similarly, for our purposes $\bar{F}$ may be defined as

$$\bar{F} \cong \left[\frac{\Delta p}{(\gamma_{LV} \cos \theta + Z)\left(\frac{f_1 t_1}{K_1}\right)^{\frac{1}{2}} \left(\frac{f_1}{1-f_1}\right)}\right] \qquad (30)$$

where in equations (29) and (30), the term, t, is the Kozeny rock textural constant, and Δp is the pressure drop.

Figure 8 shows a numerical solution of equation (26) for the case in which equal porosities, cross sectional areas, connate water saturations (equal to 0.2), and residual oil saturations (equal to 0.3) were assumed. As expected, the recovery efficiency, G, is diminished as the permeability ratio decreases; moreover, high values of G are associated with high values of $\bar{V}$ and low values of $\bar{F}$. Indeed, low values of $\bar{V}$ (less than unity) lead to instability of the interface, and to viscous fingering bypassing, discussed in the next section of this paper.

In the foregoing analyses, gravity effects have been neglected and it also has been tacitly assumed that $\bar{F}$ is always greater than $(-\bar{R}/1+\bar{R})$. The reasonableness of the latter limit was discussed by Rose and Cleary (1958), and is based on the idea that the capillary pressure at the flood front is not so great as to mean that a large negative pressure gradient is acting to oppose the forward motion of the flood front. If these factors were to be considered, then a modified form of equation (26) could be developed by the methods already presented. On the other hand, equation (26) can be generalized to apply to multilayered sands, each having a different permeability and other different characteristics, according to

$$G = \frac{a_{i+1} f_{i+1} \; (1-S_w-S_{OR})_{i+1} + \sum^{i} M_i f_i a_i (1-S_w-S_{OR})_i}{\sum^{i+1} a_i f_i (1-S_w)_i} \qquad (31)$$

where i-layers are assumed to exist, each having a different porosity, connate water saturation, cross sectional area, residual oil saturation, and permeability that is less than the permeability of the (i+1) layer.

In conclusion, it should be noted that equation (26) applies only to waterflooding conducted under conditions of constant pressure drop; hence, no quantitative check can be made of the data of Jordan et al. (1957) or the data of Richardson and Perkins (1957) who studied constant rate cases.

Capillary Controlled Rate Effects

The foregoing analyses suggest that capillary forces can play a dominant role in the recovery of oil. This is especially true of the waterflooding process, because three mobile phases (water, oil, and free gas) may coexist at the flood front. There is thus more surface energy, and more change in surface energy with time as the interfacial surface areas are altered during the recovery process, than characterizes the essentially two-phase recovery processes such as natural water drive, depletion drive, and gas-cap expansion drive.

It is not surprising, therefore, to find that the opposition of several driving forces (such as capillary, gravity, and/or displacement pressure forces) should introduce a rate sensitivity into the recovery process. This, in fact, was the position of Moore and Slobod (1956), who confirmed the intuitive suspicion of many former workers.

Of course it would be desirable to estimate quantitatively the degree of rate sensitivity that characterizes the waterflooding process, but the subject is too complex to be discussed in the present paper. Instead, I shall enumerate those effects that might be a cause for observing rate sensitivity, and make some qualitative estimate of the expected trend.

First, experience shows that the advancing contact angle is frequently observed to be greater than the receding contact angle in a way that is rate dependent (Elliott and Leese, 1957; Freundlich, 1922). Indeed, at high rates of injection the advancing contact angle might become so large that it thereafter prevents spontaneous capillary imbibition in accordance with the indications of equation (14), as applied to figure 5. Any increase in oil recovery normally associated with capillary imbibition (see below) therefore would be lost.

Similarly, it should be apparent from the form of the capillary pressure curve that when the advancing contact angle exceeds 90° due to the high rates of advance, the displacement pressure force will not suffice to cause the water (now effectively a nonwetting phase) to advance in the smaller pores. In such a case, oil contained in these smaller pores would be bypassed and lost from recovery. In this case, also, it may be imagined from geometric perspective that dF/dS is greatest when water enters the larger pore spaces. The recovery process will favor this course because the minimization of the surface energy is a contributing driving force and again there will be a rate-sensitive tendency for oil in smaller pore spaces to be bypassed.

When the contact angle is less than 90°, it is clear from equation (14) that capillary imbibition will continue even if water injection and oil production are discontinued. Both favorable and harmful effects may result from such an interruption. For example, imbibition of water during the interim period might serve to expulse oil from the tight zones of layered sands, or from the "unswept" regions. Such oil otherwise might be bypassed.

The harmful consequences of interrupted production that might occur are depicted in figure 6. A drainage of water is shown occurring behind the flood front during the shut-down period in response to the tendency for capillary imbibition to occur ahead of the flood front where the water saturation is low. The capillary pressure diagram in figure 6 can be used to explain why the water saturation ahead of the flood front does not ultimately increase to the same value to which the water saturation behind the flood front decreases. This, of course, is simply an expression of hysteresis in the capillary pressure-saturation relationship. Nevertheless, one effect of the increased water saturation ahead of the flood front might be to cause large bypassing of oil, because the relative permeability to water is increased while the relative permeability to oil is decreased ahead of the flood front. In any case, these effects would be most serious in the tight zones of layered sands, which are in fact the zones from which it is most difficult to extract oil, even in the absence of interrupted production.

Another possible rate effect associated with the existence of finite capillary forces concerns the unstable flood-front interface that develops when the mobility of the displacing phase (water) exceeds that of the displaced oil. This general problem is discussed in the next section of this paper. It is necessary to note here only that the generation of the unstable interface involves doing work, as the energy of the system is being increased (that is, the surface area, A_{WO}, is being increased). It is therefore safe to conclude that such an unstable interfacial configuration would not develop spontaneously, and, moreover, that low injection (and/or production) rates would minimize its development. And, of course, the less viscous fingering, the less the amount of bypassed oil.

The pore-doublet theory analogy as it pertains to the rate sensitivity of waterflooding in layered sands of differing permeability has been discussed earlier herein. Evidently, where the pore doublet analogy holds, low flood rates are advantageous, at least in the absence of gravity effects.

Jordan et al. (1957) have introduced the interesting, but unverified, idea that the microscopics of displacement in all cases are concerned with only the magnitude of capillary forces that exist and act on the individual water-oil interfaces at the flood front. This notion is derived from the misleading viewpoint that the capillary pressure gradient is always enormous because it must be measured across the infinitesimal thickness of the interface. The same authors note that the displacement pressure gradient is always comparatively smaller.

In point of fact, the situation is analogous to that of an engine pulling a car on a track, with the lead engine, car, and track being pushed by still another engine riding on an underlying track and pulling a series of cars. The car between the engines represents the interface, the lead engine represents the capillary imbibition force, the pushing engine represents the force delivered by the pressure of the injected water, and the trailing cars represent the viscous resistance opposing the displacement process. Clearly, the car moves as long as one of the forces is positive, even though the other may be zero, and either driving force has to overcome the same viscous resistance before motion can occur.

Consequently, in order to determine the relative effect of each driving force on the interfacial movement, we must compare the capillary pressure at particular interfaces with the gross pressure drop across the displacement system. When this is done it is seen that the displacement pressure force frequently is larger than the capillary force, so that the interfaces tend to be pushed (rather than drawn) as the flood front moves ahead. Of course, the capillary forces work to keep the interfacial surface energy of the system at a minimum, but, when the displacement pressure and flood rate are comparatively high, they will tend to control the direction of interface movement.

Looking at this question in another way, we may derive from equation (1) the rate of flood front advance, dx/dT, (for a linear flow system where water is displacing oil)

$$dx/dT = \frac{(K_O K_W)\ (\Delta P + P_c)}{K_O \mu_W x + K_W \mu_O L - K_W \mu_O x} \qquad (32)$$

where x is the position of the flood front at any time, T; μ and K are, respectively, the water and oil permeabilities and viscosities; P_c is the capillary pressure existing at the flood front; and ΔP is the pressure drop (the difference in the terminal pressures). It is clear from equation (32) that the flood-front motion and direction will be controlled by the summation of ΔP and P_c, rather than by the summation of pressure gradient and capillary pressure gradient terms.

In contrast to the interpretation of equation (32) given above, Jordan et al. (1957) speak of comparing the pressure gradient, dp/dx, with a capillary pressure gradient defined unrealistically as the capillary pressure across particular interfaces divided by the quasi-infinitesimal thickness of these interfaces. This is based on a physically unreal concept suggesting that nearly infinite capillary pressure gradients always exist at the flood front, and therefore controlling the microscopic direction of interface movement. More properly speaking, capillary pressure gradients should include a reference to the slope of the capillary pressure curve that relates a capillary pressure gradient to a saturation gradient. This, in fact, has been done implicitly in connection with equations (22) and (23) above.

Possibility of Viscous Fingering

When a viscous fluid such as oil is displaced by water in porous media, an unstable interface at the flood front is likely to develop if and when the mobility of the replacing water is greater than that of the displaced oil. For the general case of steady flow upward, Saffman and Taylor (1958) derive from equation (1) two conditions:

$$(a)\quad \left(\frac{\mu_W}{K_W}\right)-\left(\frac{\mu_O}{K_O}\right)\bar{v} + (\rho_W - \rho_O) > 0$$

which implies a favorable mobility ratio, and hence a stable interface; and

$$(b)\quad \left(\frac{\mu_W}{K_W}\right)-\left(\frac{\mu_O}{K_O}\right)\bar{v} + (\rho_W - \rho_O) < 0$$

which implies an unfavorable mobility ratio and the likelihood of an unstable interface. Thus, if gravity forces can be disregarded (for example in horizontal flow, or when the densities of the two phases are equal), one expects instability of the interface when the mobility ratio, $(K_O\mu_W/K_W\mu_O)$, is less than unity.

Interfacial instability results in a viscous fingering where the displacing phase moves irregularly into the region occupied by the phase being displaced, thus causing a bypassing and a trapping effect that tends to decrease greatly the displacement efficiency. This is different from the mobility ratio effect first presented by Rose and Witherspoon (1956), who showed that there was a decrease in displacement efficiency in layered media of different permeability zones. Both effects, however, result in decreased displacement efficiency and the amount of decrease is determined by the magnitude of the difference between the mobility of displacing and displaced phases.

Saffman and Taylor have demonstrated by model experiment that the finger pattern advance will be steady for certain steady-state flow conditions (constant rate cases) if there is continuity of pressure at the interface. They also show that discontinuities in pressure at the interface, due to interfacial tension, alter the mechanism of finger growth in a way that is rate sensitive.

In the experiments of Richardson and Perkins (1957) and Jordan et al. (1957), capillary forces exceeded the injection pressure force by a factor of two to five. An unfavorable mobility ratio condition also existed. As these were presented as accurately scaled experiments representative of prototype field conditions, it is apparent that one must consider the possibility of a rate effect on viscous fingering, and hence on oil recovery by waterflooding. The exact magnitude and mechanism of viscous fingering evidently are still to be elucidated.

Of interest in these connections is the work of van Meurs (1957) in which he observed viscous fingering effects in transparent porous media. Results from the scaled laboratory model indicated that oil recovery decreased considerably (from 88 percent to 52 percent) in homogeneous sands when the viscosity ratio between oil and water was increased from unity to 80. However, although he found stratification appreciably influenced recovery efficiency for stable interface situations, a diminished effect was noticed when the viscosity ratio was unfavorable. Apparently the two effects are not additive.

SUMMARY OF POSSIBLE CURTAILMENT EFFECTS

Any oil recovery process, including waterflooding, is extremely complex. Microscopically, the reservoir system is characterized by as many as three immiscible fluids coexisting within the irregular interstices of the porous continuum. Forces arising from surface energy, injection pressure, gravity, concentration gradients, or similar sources cause motion and fluid transfer in ways that cannot exactly be described. Macroscopically, these disturbances are known to cause (in some cases) the gross displacement and recovery of a portion of the reservoir oil.

From a mechanistic point of view, it is hardly to be expected that waterflooding, or any recovery process, will be independent of rate. It would appear, too, that the degree of rate sensitivity remains an undetermined variable until each set of conditions that characterize different reservoirs is examined in detail.

Others (Jordan et al., 1957, and Buckwalter et al., 1958) have suggested variously that recovery efficiency in waterflooding is increased, decreased, or unaffected by flooding rate. The last assertion is sometimes made with the limitation that the rate independence holds at least over the range of conditions normally expected in field operations. All these viewpoints appear to be oversimplifications even though they may have incidental validity in isolated cases.

Examination herein of the fundamental nature of capillary forces and how they might influence the recovery process, a subject largely neglected by previous investigators, leads to the conclusion that until surface energy effects are better understood, it would be premature to state that such effects can be ignored or that performance can be predicted without taking quantitative account of the magnitude and direction of the capillary forces.

In equation (1) the potential, V, was identified as the vectorial sum of the displacement pressure gradient, the hydrostatic (gravity) pressure, and the capillary pressure. The first term is related to the boundary pressures at the injection and production well bores; the second is a consequence of the differential densities of the immiscible fluids saturating the reservoir rock; and the third is proportional to the curvature of interfaces between immiscible fluids arising from the existence of free surface energy. If we limit our attention to two-dimensional flow, disregarding gravity, we notice that during curtailments in production the pressure gradient quickly approaches zero, at a rate determined by the fluid compressibilities and viscosities. The capillary pressure, however, continues to operate as a driving force, completely unaware, so to speak, that a shut down has occurred. This can lead either to an increase or a decrease in ultimate recovery, depending on various conditions discussed above in the section on "capillary controlled rate effects."

If low rates are used in the recovery process, the following benefits might possibly be obtained:

(1) There might be less bypassing of oil in the tight sections of layered sands.

(2) There would be a more efficient utilization of <u>in situ</u> reservoir energy, although this point alone does not necessarily mean that oil recovery would be increased.

(3) The operating cost (for injection pumps and so forth) might be lessened.

(4) There would be more time to study the reservoir performance and to alter operating plans (for example, for the relocation of injection and production wells to achieve greater sweep efficiency), but there is no <u>a priori</u> indication that such revisions would be beneficial.

(5) There might be a more efficient flushing of low-permeability sand lenses surrounded by a high-permeability pay section.

(6) Apparently recovery loss would be minimized due to viscous fingering.

(7) There would be less tendency for the advancing contact angle to approach 90°, a condition that diminishes the effectiveness of capillary imbibition.

Conversely, if high rates are used in the recovery process, the following alternate benefits might be obtained:

(1) The higher energy input per unit time (power) tends to do work faster. Therefore, disregarding efficiency factors, high injection rates are beneficial.

(2) If there is a tendency to maintain funicular continuity of the oil down to the point of lower residual oil saturation, high rates might effectively dislodge some of the residual oil globules after continuity is broken.

(3) Economic advantages might result from an initial high rate of recovery, even though ultimate recovery is decreased.

(4) High rates might minimize adverse gravity effects such as underflow (namely, by increasing the gravity sweep efficiency).

(5) Bypassing of oil in permeable zones might be minimized by minimizing water entry into tight zones having little or nor oil content.

(6) If the advancing contact angle measured through the displacing water phase is greater than 90°, as in the case of oil-wetted sands, high injection rates would give higher entry pressure for flushing oil out of the small pore spaces.

(7) High rates would tend to keep gas in solution, with consequent beneficial effects on the mobility of the oil phase in the vicinity of the flood front. This would minimize the harmful effects of viscous fingering. (Jordan et al., 1957, indicate that the presence of free gas has little or no effect on increasing waterflood recovery, contrary to earlier indications.)

(8) A reduction in pumping costs might be associated with high injection rates because of the increased possibility of maintaining high pressures at the production wells.

Such are some of the interesting possibilities that future workers must evaluate quantitatively. It is not a case of "pay your money and take your choice"; rather, the question of how curtailments affect waterflood recovery must be answered according to established analytic techniques to get an explicit solution of an engineering problem.

CONCLUSIONS

It is likely that oil recovery by waterflooding is influenced by rate. The recovery process is controlled by as many as three major driving forces that act together or in opposition - hydrostatic forces, hydrodynamic forces, and capillary forces. When rate is changed or when production is interrupted, the resultant of the three controlling forces changes, thus influencing the amount of oil recovered. Curtailments must be carefully programmed or otherwise recoverable oil will be lost.

At low rates, the ever-present gravity force field will have more time to act to separate immiscible fluids of unequal density, causing transfer of fluid elements from high to low potential energy levels. Other things being equal, this transfer will stabilize upward-moving water fronts and downward-moving gas fronts when the oil has intermediate density - an effect that would increase the displacement efficiency in heterogeneous, or stratified, sands.

On the other hand, if the sands are thick, homogeneous, and essentially horizontal, increased recovery would be promoted through minimizing the gravity segregation tendency (the "under flow effect"), for example, by employing high rates of displacement. If these horizontal sands are stratified, the rate effect is of variable importance, depending upon the spatial location of the high versus the low permeability layers. Thus, the gravity sweep efficiency and, therefore, recovery are determined by rate as well as such factors as density difference between the immiscible fluids, comparative magnitude of other driving forces, or permeability.

Although changing rate (or interrupting production) does not alter the gravity force field, the hydrodynamic force field is altered almost immediately in a manner related to porosity, permeability, fluid viscosity, and compressibility. On the other hand, the capillary forces generally remain constant when injection rate is changed but if they are altered they tend to increase as rate is decreased, reaching a maximum value when the rate goes to zero. As noted above, this capillary force rate dependence is a consequence of contact angle hysteresis. In any case, the resultant driving force is highly rate sensitive, because of the variable effect of rate on the hydrodynamic versus the hydrostatic versus the capillary forces.

The action of capillary forces on recovery has been especially emphasized in this paper. It has been shown that if capillary forces are acting alone, in the absence of gravity and with a zero imposed pressure gradient resulting from zero fluid-injection, there will be a more even capillary imbibition of the displacing phase into the different permeability zones of layered sands. There will also be a minimum in the advancing contact angle, and therefore a maximum in the capillary imbibition force. In such cases, low rates seem to favor increased recovery. Other situations have been described in this paper, however, where capillary imbibition would effect a bypassing and trapping of oil. It is clear, therefore, that no rule of thumb can be formulated about the effect of rate on recovery except in special cases where all initial and boundary conditions are specified.

Finally, this paper has discussed the viewpoint that low rates are desirable when the mobility ratio is unfavorable, that is when the injected water is less viscous than the replaced oil; and that low rates help to flush oil out of tight sand lenses and the unswept portion of the steady-state flood pattern. Sometimes, however, the pore surfaces may be preferentially oil wetted, and then high rates are required to force the displacing water into the small pores containing recoverable oil.

The specification of the hydrostatic and hydrodynamic force fields acting upon a given fluid element contained in a porous continuum is simple and straightforward. The theoretical discussion of surface energy given in this paper should be useful in specifying directions and magnitudes of the less well understood capillary forces. The equations show that the change in the free energy of the system per unit area (with the change in time, position, and/or saturation) is a well defined function of interfacial surface area terms multiplied by the corresponding interfacial tension terms. Thus, capillary imbibition ceases when the energy of the system has reached minimal values (dF equals zero). In any case, this paper indicates that to make quantitative use of surface energy concepts it is imperative that the magnitude of interfacial surface areas, especially those of the fluid-fluid type, be measured.

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