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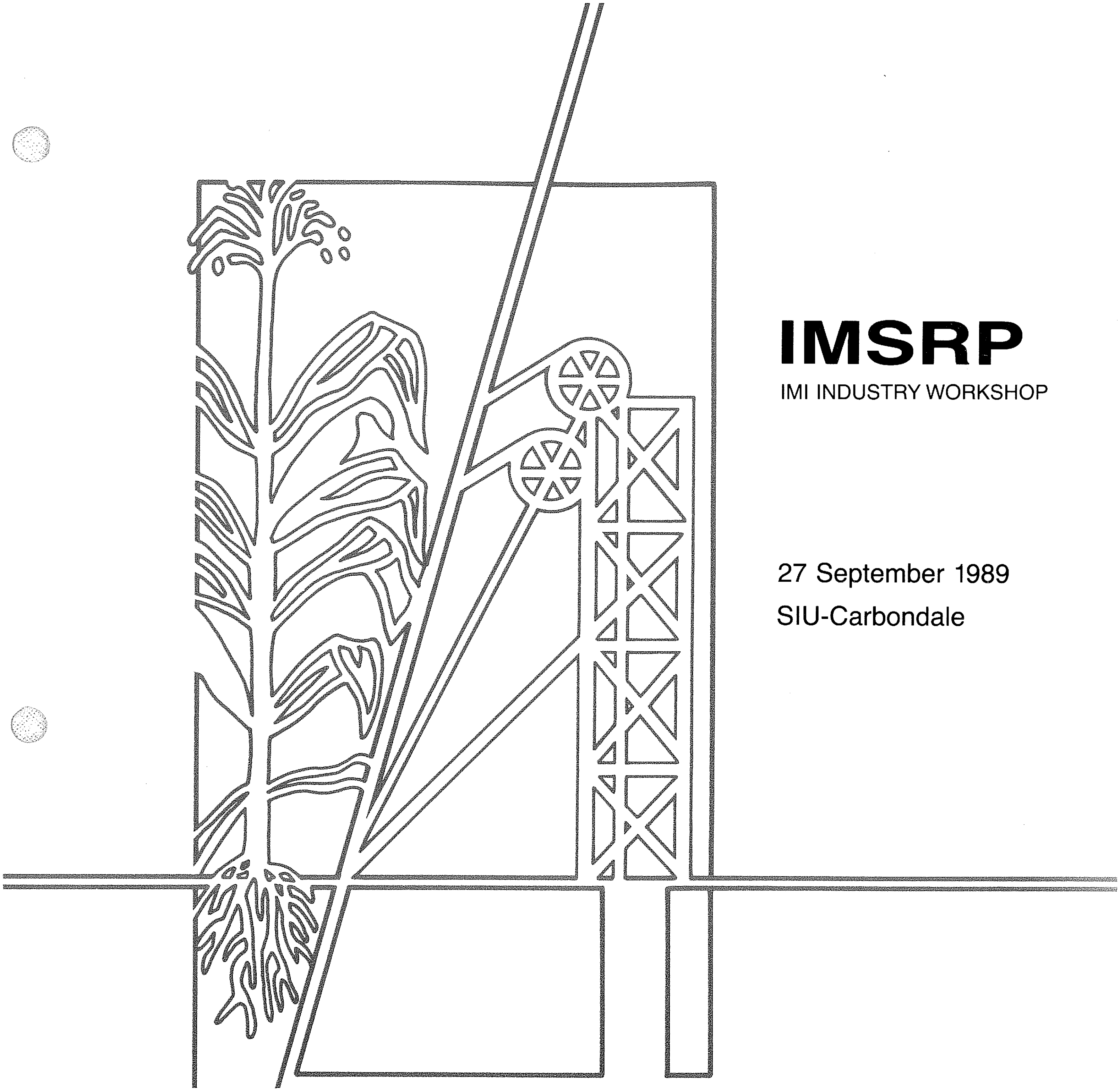

SIU Subsidence Prediction Model for Partial Extraction Room-and-Pillar Mining

This work was sponsored by the Illinois Mine Subsidence Research Program administered by the Illinois State Geological Survey through a grant from the Illinois Coal Development Board of the Illinois Department of Energy and Natural Resources and members of the Illinois Coal Association and Illinois Farm Bureau.

SIU SUBSIDENCE PREDICTION MODEL
FOR PARTIAL EXTRACTION ROOM-AND-PILLAR MINING

Department of Mining Engineering
Southern Illinois University at Carbondale
September 27, 1989

workshop conducted by
Dr. Y. P. Chugh and Dr. W. M. Pytel

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SIU Subsidence Prediction Model for Partial Extraction Room-and-Pillar Mining

PANEL.2D

USER'S Manual

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LIST OF SYMBOLS

b_i	-	relative width of the i-th layer of overburden beam
B	-	pillar width, in
B	-	plate size, in
E	-	modulus of elasticity, psi
E_i	-	deformation modulus of the i-th overburden stratum, psi
E_1	-	elastic parameter of the Kelvin-Voigt unit of standard Burger model, psi
E_2	-	elastic parameter of the Maxwell unit of standard Burger Model, psi
EI	-	flexural stiffness of a composite overburden beam lb in^2/in
f(t)	-	creep function for Burger model
fe	-	reduction factor for modulus of deformation due to size effect
GF	-	shear stiffness of a composite overburden beam, lb/in
h(z)	-	function describing variation of the vertical displacement with depth
h_i	-	thickness of the i-th overburden stratum, in
H_c	-	thickness of weak floor strata, in
H_p	-	pillar height, in

I	- moment of inertia, in^4
I_{pl}	- plasticity factor
I_s	- shape coefficient
l	- constant for Vlasov model of weak floor strata
L	- pillar length, in
m	- number of layers within overburden
n	- number of pillars in a panel
N_1	- viscous parameter of the Kelvin-Voigt unit of the standard Burger model, psi day
N_2	- viscous parameter of the Maxwell unit of the standard Burger model, psi day
q	- external pressure, psi
q_0	- pillar pressure after mining, psi
q_v	- pillar pressure before mining, psi
q_{ult}	- ultimate bearing capacity, psi
r_i	- distance between the neutral axis and the center of the i-th overburden stratum, in
t	- time, days
u	- horizontal displacement, in
w	- vertical displacement, in
x, y, z	- coordinates, in
α_1, α_2, α_3	- dimensionless parameters for the standard Burger model
ε_{el}	- pillar strain due to elastic behavior

ε_{pl} - additional pillar strain due to plastic behavior

ν - Poisson's ratio for weak floor strata

ν_i - Poisson's ratio for i-th overburden stratum

1. INTRODUCTION

A two-dimensional subsidence prediction model for partial extraction room-and-pillar mining (PANEL.2D) was developed and validated under grants from the Mine Systems Design and Ground Control Generic Center, Illinois Mine Subsidence Research Program, and the U.S. Bureau of Mines. The theoretical background for the model and some of the validation results have been presented by the authors elsewhere (Pytel, Chugh, Zabel, Caudle, 1988; Pytel and Chugh, 1989). This manual has been prepared to permit engineers and technologist to use this model. Copies of Pytel et al (1988) and Pytel and Chugh (1989) are included in Appendix I to provide the reader with theoretical background for the model and validation results.

2. OVERVIEW OF THE MODEL

PANEL.2D is an approximate analytical model based on the theory of beams resting on elastic or inelastic foundations. The physical problem (Figure 1a) consisting of overburden, coal pillars, and floor strata is idealized as in Figure 1b and analyzed approximately. Overburden strata are idealized as a composite elastic beam of unit width with stepwise varying flexural and shear stiffness. The loads imposed on the composite beam are transmitted to weak floor strata through coal pillars with elastic or elastic-plastic behavior. Coal pillars are represented by one-dimensional springs with linear or non-linear characteristics. The floor strata underneath the pillars is idealized as a two-layer rock mass with a weak upper layer resting on a non-deformable rock layer. The weak upper layer represents all weak floor strata as a single homogeneous, isotropic layer with equivalent elastic and time-dependent deformation behavior. Stress-deformation behavior of weak floor strata may be represented by one of several models such as elastic-half space, the Winkler model, confined clay layer, or Vlasov elastic layer (Pytel et al, 1988).

It is assumed that vertical bed separations do not occur in overburden strata due to small mining deformations in room-and-pillar mining. However, lateral movements along different beds may occur during the bending process of overburden strata, i.e. roof-coal-floor strata interfaces may be assumed rough or smooth. Imposed external loads on the beam are assumed to be gravitational only due to the weight of overburden. These loads induce only vertical reactive forces at the pillars (Figure 1c).

The overburden beam, coal seam, and floor strata are subdivided into small blocks through grid network. The size of the grid block depends upon the desired calculation accuracy or the size of pillars and openings. One or more panels may be considered in the analysis process.

Capabilities of the Model

1) All openings and pillars in one or more panels up to a maximum of 120 pillars may be included in the model.

2) The program permits three options in load-deformation behavior of coal measure rocks:

   (i) Linear elastic behavior of all strata including overburden, coal seam, and floor strata.

   (ii) As in (i) above and time-dependent behavior of weak floor strata only.

   (iii) As in (i) above and elastic-plastic behavior of coal pillars.

3) Time sequence of mining can be modeled in two-dimensions.

4) Permits consideration of varying overburden lithology and floor strata lithology.

Limitations of the Model

1) Small panel depth to panel width ratio which justifies application of the beam theory.

2) Model formulation assumes small strain theory.

3) In-situ horizontal stresses cannot be simulated.

4) This model does not permit bed separations.

5) Non-homogeneous floor strata can be modeled as a two-layer system with the weak layer overlying a non-deformable layer.

6) Only two-dimensional problems may be solved.

7) It requires a regular panel geometry and mining sequence in the plane perpendicular to the paper.

3. COMPUTER SYSTEM REQUIREMENTS

The PANEL.2D program has been written in FORTRAN language and is presently operational on a PRIME 9955 mini-computer in the College of Engineering and Technology with a PRIMOS V.21.0.5 operating system. The program may be modified to run on a PC, but the size of problems that may be run will be significantly reduced. The program output may be analyzed for graphical output through the GRAPH.2 option (Missavage, 1987).

4. MODELING OF THE PHYSICAL PROBLEM FOR PANEL.2D ANALYTICAL MODEL

4.1 Modeling of Overburden Strata as an Equivalent Beam

It is reasonable to assume that in a partial extraction room-and-pillar mining system at least the overburden remains in a linear elastic deformation phase. The overburden can then be transformed into a composite beam of unit width and with stepwise varying fexural stiffness EI(i) [$lbin^2/in$] and shear stiffness GF(i) [lb/in] reflecting deformability characteristics of roof strata along a given section "j". The stiffness of overburden strata beam depends on the degree of bonding between "m" layers and two extreme cases are considered in the model:

I. Different overburden layers are fully bonded and the overburden is treated as a one thick beam of stiffness EI(j) or GF(j) (see Figure 2):

$$EI(j) = E_m \sum_1^m b_i h_i^3/12 + E_m \sum_1^m b_i h_i r_j^2 \quad (1)$$

$$GF(j) = E_m \sum_1^m b_i h_i/2(1 + \nu_i) \quad (2)$$

II. Overburden strata interfaces are perfectly smooth, and the overburden beam consists of a set of sub-beams which may be characterized by the following stiffness:

$$EI(j) = E_m \sum_1^m b_i h_i^3/12 \quad (3)$$

$$GF(j) = E_m \sum_1^m b_i h_i/2(1 + \nu_i) \quad (4)$$

where E_i and ν_i are the elasticity modulus and Poisson's ratio respectively of the (i)th layer of h_i thickness and relative width $b_i = E_i/E_m$ (E_m is the deformability modulus for the m-th layer). A small computer program (OVER) was developed in the FORTRAN language using the above mentioned relationships (1-4). The following numerical example illustrates the calculations for beam stiffness.

Numerical Example: The overburden thickness of 379 ft. consists of 30 different rock layers. The overburden lithology with thickness, flexural modulus and Poisson's ratio for different layers, is presented in Table 1.

The input data (file: IN.OVER) consists of three columns of m (number of layers m = 30) components: flexural modulus [psi], layer thickness [in] and Poisson's ratio (see Table 2).

Program "OVER" is invoked by typing:

x OVER

The results of calculation (file: OUT.OVER) are displayed by a command

em OUT.OVER

and the display shows the flexural and shear overburden stiffness for bonded and unbonded interface cases (see Table 3).

4.2 Modeling of Floor Strata

The immediate floor strata is transformed into an equivalent homogeneous rock mass of thickness H_c. Depending upon the ratio of pillar width B and thickness H_c of floor deformable strata, the following linear models of weak floor strata behavior are available in the program:

1. Winkler model - ($B/H_c > 2.0$) which assumes that the deflection w(x,y) of the rock/soil medium at any point on the

surface is directly proportional to the stress q(x,y) applied at that point and independent of stresses at other points.

2. The elastic half-space - ($B/H_c < 0.25$) for which the deflection of the weak floor strata surface can be obtained by integrating Boussinesq's solution for the concentrated force acting on the surface of an isotropic elastic half-space. Settlement value depends on the shape of the loaded area and on the distance from the center of the loaded area to the given point.

3. Confined clay layer - ($0.25 \leq B/H_c \leq 2.0$) developed by Taylor and Matyas (1983). This model considers the elastic settlement of a uniformly loaded strip area on clay foundation. The calculation is based on the solution of Kelvin's equation for a line load acting within an infinite solid.

4. Vlasov model of an elastic layer - ($0.25 \leq B/H_c \leq 2.0$). It assumes a state of strain in the foundation which imposes the following relationships for horizontal and vertical displacements:

$$u(x,z) = 0 \ , \ w(x,z) = w(x)h(z) \tag{5}$$

where h(z) is an arbitrarily assumed function describing the variation of the vertical displacement w(x,z) with depth (z), and may be expressed as:

$$h(z) = \sin h \ [\gamma(H_c - z)/l]/\sin h \ (\gamma H_c/l) \tag{6}$$

The qualitative differences between the above-mentioned time-independent (elastic) models of weak floor strata behavior are illustrated in Figure 3. It should be emphasized that all these models depend on two parameters only:

. Poisson's ratio ν which may be determined from laboratory tests on rock cores, and

. Modulus of elasticity E of the rock determined from plate load test using standard procedures and scale (size) effect factor $f_e = 0.3 - 0.6$ suitable for plate sizes $B_{pl} = 6\text{-}12$ [in], Chugh, Pula, and Pytel (1989):

$$E = \frac{q}{w} B_{pl} (1-\nu^2) I_s f_e \tag{7}$$

where I_s is the coefficient depending on the plate shape (for rigid square plate $I \approx 0.88$), B_{pl} is the plate size, and q is unit pressure on rock/soil (it is accepted $q \approx 0.5\ q_{ult}$), w is the corresponding plate settlement, and q_{ult} is the ultimate bearing capacity.

Additional time-dependent displacements of the mining structure are assumed to be related to visco-elastic behavior of weak floor strata only. For an isolated footing (test plate, pillar) subjected to time-independent load (q), the settlement may be approximately expressed as follows:

$$w(t) = w(0)\ f(t) \tag{8}$$

where w(0) represents the immediate settlement (or elastic solution) and function f(t) reflects visco-elastic weak floor strata behavior for time t. In the program PANEL.2D, the standard Burger rheological model, (Figure 4) is used to represent weak floor strata for which

$$f(t) = 1 + \frac{E_2}{E_1}[1 - \exp(- \frac{E_1}{N_1} t)] + \frac{E_2}{N_2} t \tag{9}$$

where E_1, E_2, N_1, N_2 are elastic and viscous parameters of the model. These parameters may be obtained using the following approaches:

1. Plate Loading Tests - Time-dependent plate loading tests may be conducted to obtain time-dependent deformations of weak floor strata at a constant load. The data can be analyzed to estimate E_1, E_2, N_1, N_2 parameters using available computer models (Hardy and Wang, 1969), or other simpler techniques.

2. In-mine convergence measurements are very suitable for determination of E_2/N_2 based on long-term deformations. If convergence points are installed immediately after mining, the parameter N_1 may also be estimated. The parameter E_2 may be estimated from plate loading test data. It is important to note that parameters N_1, E_1, and N_2 determined from convergence measurements represent overall rock mass behavior associated with the opening rather than weak floor strata only. These parameters

are truly representative of the physical problem and should therefore be most relevant.

4.3 Back Calculations from Analytical Model Studies

The PANEL.2D model may be used to estimate parameters E_1, E_2, N_1, and N_2 by comparing predicted and observed deformations underground. This is the most complex method and is not recommended except in very unusual circumstances.

Numerical Example: Assuming that the plate influence extends to a depth $H = 5B_{pl}$, it can be assumed that for $B_{pl} = 1$ ft the modulus of elasticity determined from Eq. 7 reflects the equivalent deformability of weak floor strata down to a depth of 5 ft. From Eq. 7, the modulus of elasticity E may be calculated as:

$$E = \frac{600}{0.083}\, 12(1-0.35^2)\ 0.88\ 0.6 = 40.2E4 \text{ psi.} \qquad (10)$$

A typical time-dependent plate load test is shown in Figure 5. The constants $\alpha_1 = E_2/E_1$, $\alpha_2 = E_1/N_1$ and $\alpha_3 = E_2/N_2$ may be obtained from the following system of equations (Eq. 8 for various time periods) using data from Figure 5:

$$0.160 = 0.083\ [1+\alpha_1(1-e^{-3\alpha_2}) + 3\alpha_3]$$

$$0.175 = 0.083\ [1+\alpha_1(1-e^{-9\alpha_2}) + 9\alpha_3] \qquad (11)$$

$$0.176 = 0.083\ [1+\alpha_1(1-e^{-24\alpha_2}) + 24\alpha_3].$$

Solving these equations, $\alpha_1 = 1.11_1$ $\alpha_2 = 0.60$ and $\alpha_3 = 3.7E - 4$ and then the Burger model constants are:

$E_1 = 36.21E3$ psi, $E_2 = E = 40.20E3$ psi, $N_1 = 60.35E3$ psi.hour, and $N_2 = 1.086E8$ psi.hour. These parameters were utilized in the numerical example presented in Appendix II.

4.4 Modeling of Coal Pillars

It is assumed that pillars can deform according to one dimensional compression. A coal pillar is represented by a set of non-linear springs sandwiched between the overburden strata and the deformable weak floor strata. The non-linear response of the coal pillar with width to height ratio greater than six would have a central elastic core of infinite strength. The stress-strain relationship for such an elasto-plastic rectangular pillar may be expressed as

$$\varepsilon_{pl} = \varepsilon_{el}\ (1+I_{pl}) = q_o/E_p\ \ [1+\{\alpha(\beta_1+\beta_2)(2-0.5\alpha^2)+4\alpha^2+8/5\alpha^4\}/\{(\beta_1-2\alpha)(\beta_2-2\alpha)\}] \qquad (12)$$

where q_o is the actual pillar pressure after mining, E_p is the elasticity modulus of coal, and I_{pl} is a plasticity factor depending on ratios $\beta_1 = B/3H_p$, $\beta_2 = L/3H_p$ and $\alpha = q_o/4q_v$ (B and L are pillar length and pillar width respectively, H_p is pillar height and q_v is the overburden pressure before mining. The q_o pressure is obtained from the solution iterative procedure.

5. PROBLEM SOLUTION

The problem solution technique is described in more detail in Pytel et al (1988). The problem yield displacements/ settlements and reactive loads acting on all modeled pillars. The settlement data is plotted as "SUBSIDENCE" profile. This profile is differentiated once to provide "SLOPE" profile and twice to yield "CURVATURE" profile.

6. INPUT DATA

The input data involves the following variables:

Variable	Type	Description
n	integer	Total number of points to be analyzed (total number of pillars and openings)
opl 1	integer	The number assigned to the leftmost opening in panel 1
opr 1	integer	The number assigned to the rightmost opening in panel 1
opl 2	integer	The number assigned to the leftmost opening in panel 2
opr 2	integer	The number assigned to the rightmost opening in panel 2
opl 3	integer	The number assigned to the leftmost opening in panel 3
opr 3	integer	The number assigned to the rightmost opening in panel 3

For the case of only one panel, opr 2 and opr 3 are equal to one, and in the case where there are two panels, opr 3 must be equal to one.

q_o	real	overburden pressure before mining [psi]
etl	integer	should be $\geq$ 1 if et1 = 1 (only elastic behavior of pillars is assumed). if et1 = 6 (elasto-plastic behavior of pillars, 6 iterations).
sym	integer	sym = 1 if the problem is symmetric sym = 0 if there is no symmetry
ttt	real	time [sec, day, year etc.] if ttt = 0.0 viscous behavior of weak floor strata is not considered. (if et1 > 1, ttt must be equal to zero).
niu	real	Poisson's ratio for roof weak strata
nil	real	Poisson's ratio for floor weak strata
aa	real	pillar length for elastic half-space model (aa > 10B) [in], where B is the width of the pillar.
H_p	real	pillar height [in]
e01	real	elastic modulus parameter of the Kelvin-Voigt unit for weak floor strata behavior, [psi]
e02	real	viscous parameter of the Maxwell unit for weak floor strata behavior, [psi]

en1	real	viscous parameter of the Kelvin-Voigt unit for weak floor strata, [psi.day]
en2	real	viscous parameter of the Maxwell unit for weak floor strata, [psi.day]

For each of the "n" points to be analyzed:

EI (i)	real	flexural stiffness of the (i)th span (should be calculated using the "OVER" program) [lb.in]
GF (i)	real	shear stiffness of the (i)th span [lb/in]
B (i)	real	width of pillar [in]
Bl (i)	real	length of pillar [in]
e0u (i)	real	elasticity modulus for weak roof strata [psi]
e0l (i)	real	elasticity modulus for weak floor strata [psi]
delta (i)	real	vertical displacement of the weak floor surface at the i-th point due to some external, independent effects such as surcharge, [in]
H_u (i)	real	thickness of the weak roof strata, [in]
H_l (i)	real	thickness of the weak floor strata, [in]

L (i)	real	distance between pillar or opening center and the center of previous opening or pillar center [in], l(1)=0)
Lp (i)	real	length of pillar + width of opening [in], (see also Figure 7)
Ep (i)	real	elasticity modulus for coal [psi]
gam	real	constant (gam = γ from Eq. 6) for Vlasov model
vl	real	constant (vl = l from Eq. 8) for Vlasov model

7. RUNNING THE PROGRAM

After compiling and linking, the program is invoked by typing:

x panel.2d.

The next step is the information on the input data file name and to decide which weak floor strata model you wish to determine settlements and reactions. The following display will appear on the screen:

Weak Floor Strata Models

[1] Winkler

[2] Zemochkin and Sinitsyn

(Elastic Homogeneous Half-Space)

[3] Vlasov

[4] Confined Clay layer

[5]

[6] Quit

Type in a number (1-4) corresponding to the desired choice. The program will calculate settlements, reactions, surface slope and curvature based on the model chosen. Write this information in a file(s) of the user's choice, then prompt the user again for a selection. To quit, simply type 6. The typical display while running the program is shown in Appendix II.

8. OUTPUT DATA

The output data is furnished in five (5) files of names chosen by the user (see example in next section). The first file (OUTPUT.DAT) lists the number of the pillar, their settlement and reaction pressure after mining. This data is headed by model name. The next four files deal with final settlement, final pressure, surface slope $*10^4$ and curvature [1/in $* 10^6$]. These data follow the distance [ft x 1000] from the leftmost edge to a given pillar. The latter four files may be used for presentation of results in graphical form (GRAPH 2. option - by R. Missavage). The user's manual for this option will be available during this course.

9. CONCLUDING REMARKS

PANEL.2D computer model may be used to predict in-mine as well as surface subsidence movements in partial extraction room-and-pillar mining due to deformations of roof strata, coal pillars, and immediate floor strata. The model has been validated at three mines to date and has resulted in relatively accurate predictions of in-mine and/or surface subsidence movements (Chugh and Pytel, 1988; Pytel and Chugh, 1989; Chugh, Pytel, Pula, 1989). It is planned to extend this model for three dimensional analysis over the next two years. The authors would like to ask the users to report any mistakes and/or inconsistencies in the use of the model or the User's manual so that these could be improved.

REFERENCES

Chugh Y.P., Pula O., Pytel W.M., (1989), Design of Coal Pillars For Weak Floor Strata Conditions - A Rational Approach, Paper submitted for the 7th Annual Workshop, Generic Mineral Technology Center, Mine System Design and Ground Control, VPI/SU, Blacksburg, Virginia.

Hardy Jr., H.R. and Wang, Y.J. (1969), "A Computer Program for the Analysis of Incremented Creep Data in Terms of the Generalized Burger Model," Department of Mineral Engineering, The Pennsylvania State University, Internal Report RML-IR/ 69-12.

Hardy M.P. Christiansen M., Crouch S.L., (1977), Hybrid Computer System for Optimization of Extraction Procedures in Tabular Coal Deposits, Dept. of Civil and Min. Eng., Univ. of Minnesota, Minneapolis, USBM Contract Report.

Missavage R., (1987), Graph 2 Rev. 1.1, Oct. 1987, User's Manual, Southern Illinois University.

Pytel W.M., Chugh Y.P., Zabel B., Caudle R.D., (1988), A Simplified Two-dimensional Analysis of the Roof-Pillar-Floor Interaction Problem in Coal Mines, Proc. 7th Int. Conf. on Ground Control in Mining (ed. Syd. S. Peng), Morgantown, W.V., pp. 271-281.

Pytel W.M., Chugh Y.P., (1989), An Analysis of Roof-Pillar-Weak Floor Interaction in Partial Extraction Room-and-Pillar Mining, Proc. 30th U.S. Symposium on Rock Mechanics (ed. A. Wahab Khair), Morgantown, W.V., pp. 621-628.

Taylor B.B., Matyas E.L., (1983), Settlement of a Strip Footing on a Confined Clay Layer, Can. Geot. J., v. 20, pp. 535-542

Wilson A., Ashwin D.P., (1972), Research into the Determination of Pillar Size, Part I, A Hypothesis Concerning Pillar Stability, The Mining Engineer, 131, pp. 409-417.

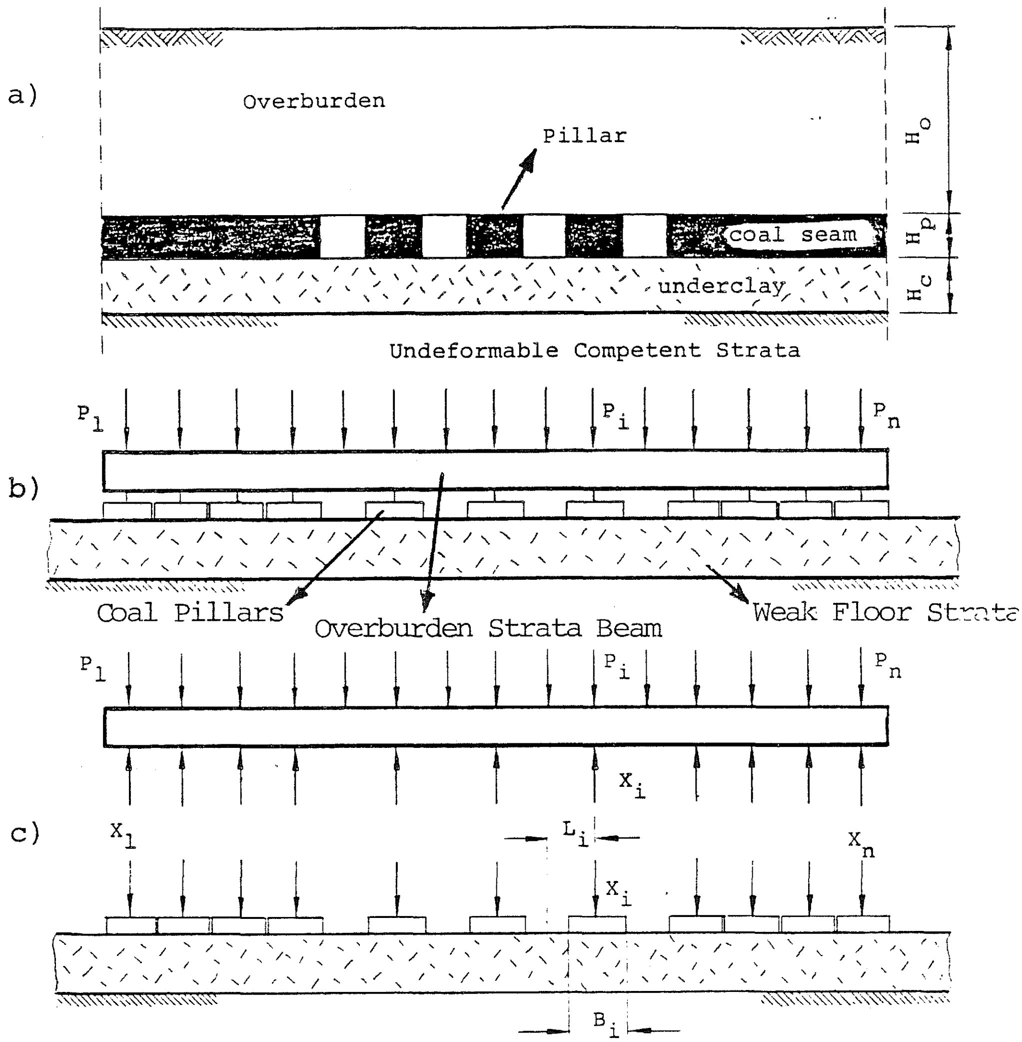

Figure 1a - Physical Problem

b - Statically Equivalent System

c - Equivalent Static Problem Showing Forces And Reactions

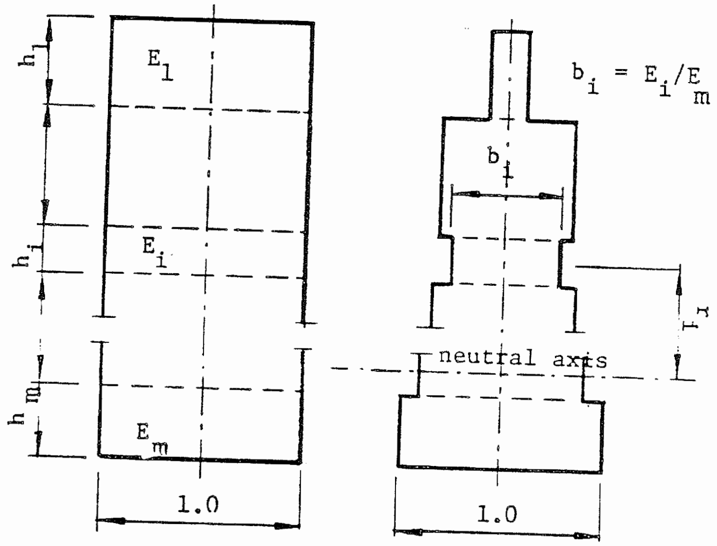

Figure 2. Calculation of Stepwise Varying Stiffness For Overburden Beam.

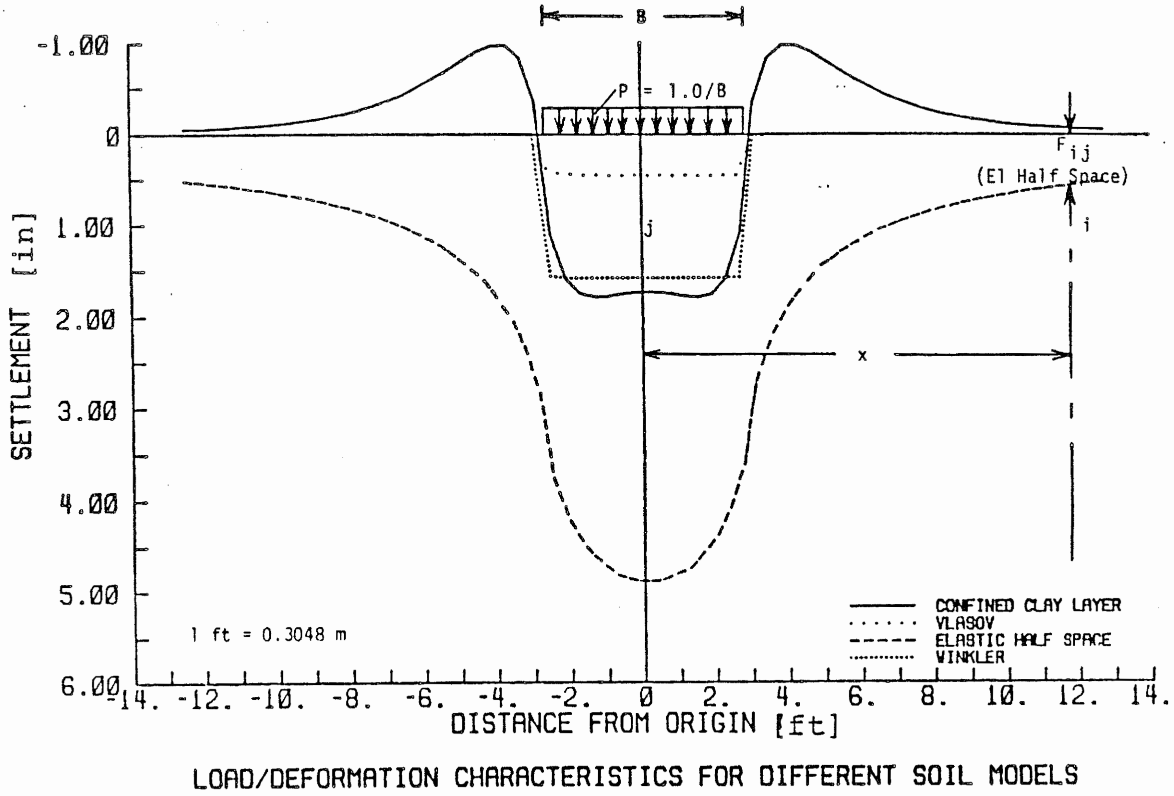

Figure 3. Surface Settlement Profiles For Different Weak Floor Strata Models.

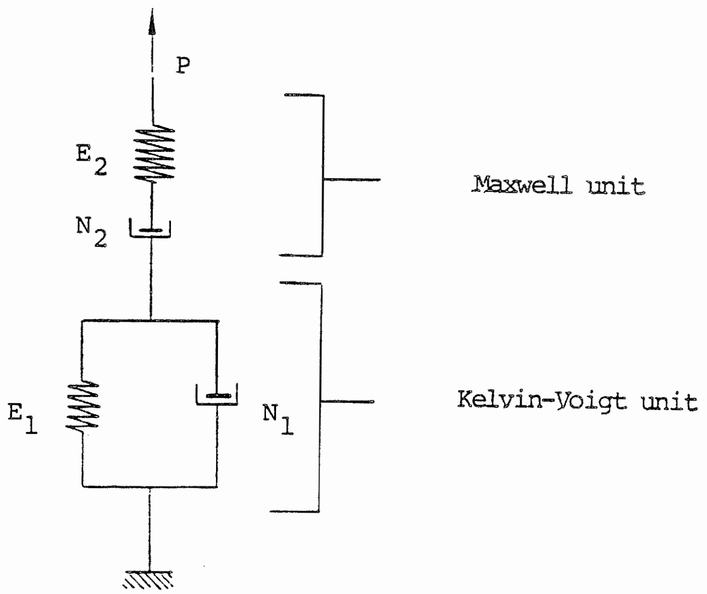

Figure 4. Burger Rheological Model.

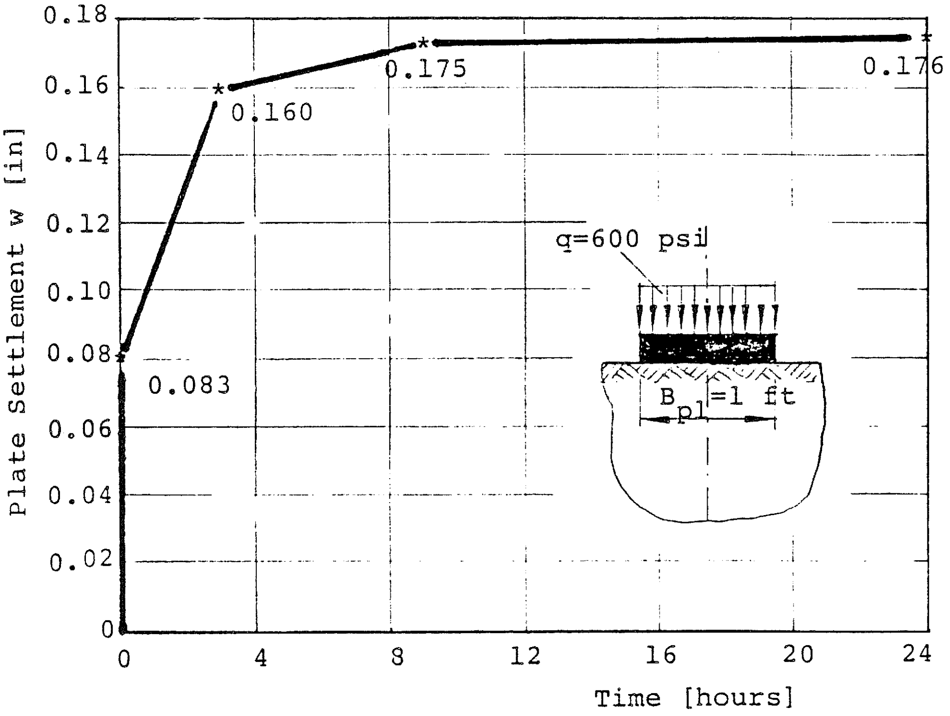

Figure 5. Plate Settlement as a Function of Time For One Load Increment.

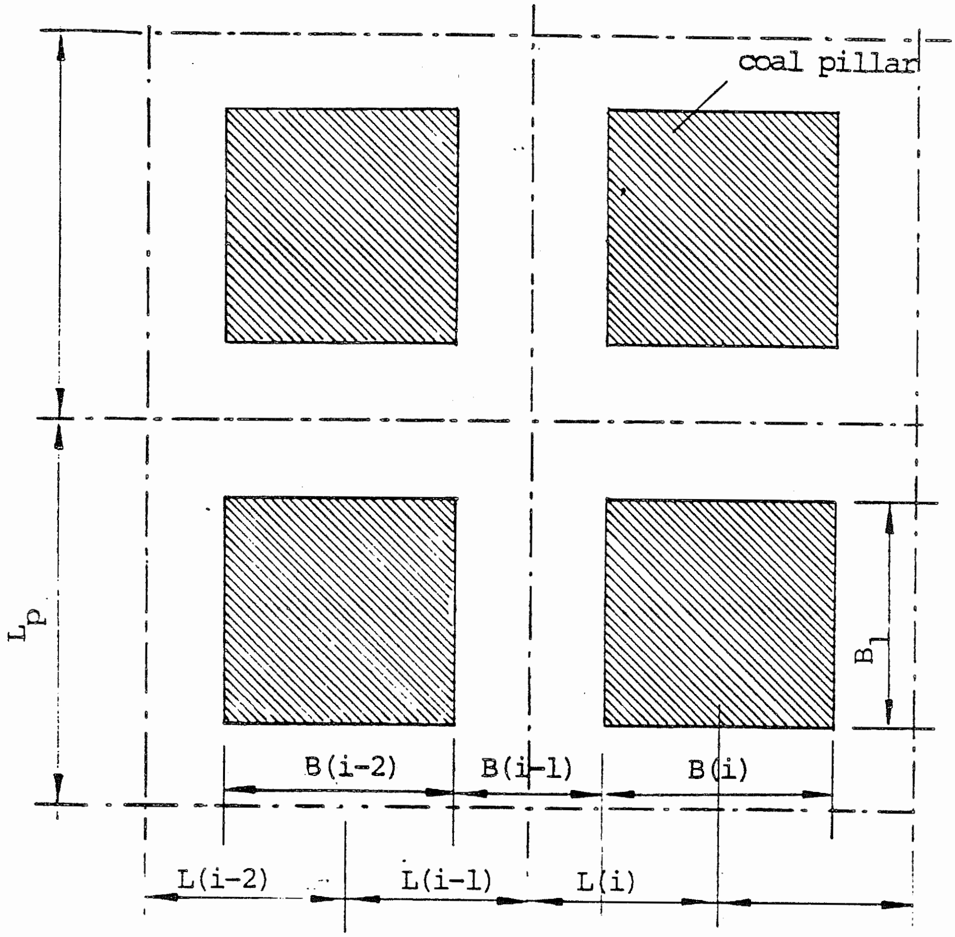

Figure 6. Dimensions in Pillar Vicinity.

Table 1. Overburden Strata Properties (Data For Numerical Example)

Flexural Modulus [psi]	Layer Thickness [in]	Poisson's Ratio	Type of Rock
2.E3	967.92	0.4	glacial deposits
18.E4	72.00	0.2	gray shale
32.E4	31.92	0.15	limestone
18.E4	229.92	0.2	gray shale
8.E4	6.96	0.3	coal
1.6E4	12.00	0.4	underclay
18.E4	186.96	0.2	gray shale
24.E4	41.04	0.25	sandstone
18.E4	147.96	0.2	gray sandy shale
24.E4	27.96	0.25	sandstone
18.E4	134.04	0.2	gray sandy shale
32.E4	31.92	0.15	limestone
32.E4	70.56	0.15	limestone
18.E4	108.00	0.2	dark gray shale
24.E4	195.96	0.25	sandstone
18.E4	186.00	0.2	gray sandy shale
8.E4	42.00	0.3	coal
4.E4	92.04	0.35	clay shale / underclay
32.E4	60.00	0.15	limestone
18.E4	66.00	0.2	gray shale
24.E4	26.04	0.25	sandstone
18.E4	96.00	0.2	gray shale
32.E4	66.96	0.15	limestone
18.E4	17.04	0.2	black shale
8.E4	54.96	0.3	coal
4.E4	185.04	0.3	clay shale
24.E4	113.04	0.25	sandstone
18.E4	312.96	0.2	gray sandy shale
24.E4	139.92	0.25	sandstone
18.E4	821.04	0.2	gray / dark shale

Table 2. Input Data File For Program OVER (File IN.OVER)

30		
2.E3	967.92	0.4
18.E4	72.00	0.2
32.E4	31.92	0.15
18.E4	229.92	0.2
8.E4	6.96	0.3
1.6E4	12.00	0.4
18.E4	186.96	0.2
24.E4	41.04	0.25
18.E4	147.96	0.2
24.E4	27.96	0.25
18.E4	134.04	0.2
32.E4	31.92	0.15
32.E4	70.56	0.15
18.E4	108.00	0.2
24.E4	195.96	0.25
18.E4	186.00	0.2
8.E4	42.00	0.3
4.E4	92.04	0.35
32.E4	60.00	0.15
18.E4	66.00	0.2
24.E4	26.04	0.25
18.E4	96.00	0.2
32.E4	66.96	0.15
18.E4	17.04	0.2
8.E4	54.96	0.3
4.E4	185.04	0.3
24.E4	113.04	0.25
18.E4	312.96	0.2
24.E4	139.92	0.25
18.E4	821.04	0.2

Table 3. Output Data File From Program OVER (File OUT.OVER)

	Bonded interfaces	Unbonded interfaces	
Case 1	EIb= 0.7273E+15	EIub= 0.9703E+13	GF= 0.2751E+09

A P P E N D I X I

7th INTERNATIONAL CONFERENCE ON GROUND CONTROL IN MINING

A SIMPLIFIED TWO-DIMENSIONAL ANALYSIS OF THE ROOF-PILLAR-FLOOR INTERACTION PROBLEM IN COAL MINES

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ABSTRACT

A two-dimensional time-dependent analysis of a overburden -coal pillar-weak floor strata interaction problem is presented as a beam model consisting of a composite roof beam resting on multiple elastic foundations (pillars) underlain by a composite rock mass representing immediate floor strata. Several different material models may be considered for the immediate floor strata. The analysis can include all openings and pillars in a panel and permits bed separations in the roof composite beam. The model has permitted identification of the relative significance of different geometric and mechanical behavior parameters which govern the system. The paper presents the theoretical background for the model as well as its application to a mine in Illinois where surface and underground geotechnical observations have been conducted over the last two years.

INTRODUCTION

Stability of mine workings as well as the characteristics of surface subsidence movements in underground mining of stratified deposits such as coal are significantly influenced by roof-pillar-floor interaction. It may manifest itself underground as pillar sloughing, floor heave, roof falls and in extreme cases as coal mine bumps, and on the surface as localized and/or trough subsidence movements. Very limited research has been done on the roof-pillar-floor interaction problems in coal mines. This research was initiated with the specific objectives to predict 1) pillar settlements and floor heave underground due to weak floor strata, and 2) associated surface subsidence movements.

Design of coal pillars under weak floor conditions in the United States is presently based on the ultimate bearing capacity (UBC) of immediate floor strata without a consideration of pillar settlements. Unfortunately foundation failure of coal pillars rarely occurs. Pillar settlements on weak floor strata, with associated floor heave in mine openings or differential pillar settlements, may result in changed geometry of mine roadways, roof, coal pillar, and floor failures and surface and sub-surface movements. Excessive pillar settlement or settlement rates may significantly increase mining cost and lead to abandonment of mining operations. Therefore it is imperative that capabilities to predict pillar settlements with a consideration of roof-pillar-floor interaction be developed. These problems are of significant interest in Illinois Coal Basin mines where coal seams are generally associated with weak floor strata which varies 1-2 m in thickness.

ANALYTICAL APPROACH AND DEVELOPMENT OF THE MODEL

The physical problem consisting of overburden, coal pillar, and weak floor strata is shown in Figure 1a. The solution involves transforming the problem into an equivalent indeterminate beam. Overburden associated with the coal seam is transformed into a composite beam with stepwise varying stiffness (flexural rigidity EI and shear stiffness). The overburden load acts on the beam as a uniformly applied load, and it is transmitted to the weak floor strata through segmented continuous footings representing panel pillars of known width (Fig. 1b). The immediate floor strata is transformed into rock mass with an equivalent constant deformability. The overall roof beam is assumed to rest on the barrier pillars which may also have variable stiffness. The present model may include all openings and pillars in a panel. The present model neglects shearing forces at the roof-coal-floor interfaces and lateral stresses in the ground. Efforts are currently underway to modify the present model to overcome the above deficiencies. The small magnitude of expected beam deflections and the small ratio of the panel depth to panel width justifies beam theory application. The model formulation assumes small strain theory.

The analytical model formulation is presented in Eq.1.

$$[\delta] \times \{X\} = \{\delta_p\} + \{\Delta\} \qquad (1)$$

In this equation $[\delta]$ and $\{\delta_p\}$ are matrix and vector of an equivalent beam displacements due to unit forces and external load $\{P\}$ respectively, and $\{\Delta\}$ is a vector of vertical foundation or pillar displacements due to other effects such as surcharge loading etc.. The matrix $[\delta]$ incorporates beam geometry, beam stiffness, geometry of mine workings, and stiffness of the immediate floor strata. The problem solution, namely the $\{X\}$ vector of support reactions is obtained by using the Maxwell-Betti reciprocal theorem (Zemochkin-Sinitsyn's method modified by Krol [1] and Biernatowski and Pytel [2]) which permits the determination of vertical support reactions in pillars from the system of algebraic, linear equations (Eq.1). A more detailed mathematical formulation of the model is presented elsewhere (Pytel [3]).

The settlement of pillars $\{Y\}$ is then calculated, using the principle of superposition, as follows:

$$\{Y\} = [F] \times \{X\} \qquad (2)$$

where matrix $[F]$ represents displacement due to unit load based on an accepted model of weak floor strata.

Linear Models of Weak Floor Strata Behavior

Winkler's Model: The deflection F_{ij} at any point (i) on the surface (component of [F] matrix) of the soil/rock medium) is directly proportional to the stress applied at that point and is independent of stress applied at any other point (j). The settlement F_{ij} (see Fig 2) due to a linearly distributed unit force can be expressed as follows:

$$\text{if } i=j \qquad F_{ij} = 1/B_j K_i$$
$$\text{if } i \neq j \qquad F_{ij} = 0 \qquad (3)$$

where K_i is the modulus of subgrade reaction of the weak floor strata, and B_j is the width of the loaded area.

Isotropic Elastic Medium: The deflection F_{ij} in this case is obtained through Boussinesq's solution for surface deflection of an isotropic, elastic halfspace subjected to a unit force distributed on a rectangular area, Zemochkin-Sinitsyn [4], Selvadurai [5]. For a linearly distributed unit force:

$$F_{ij} = \frac{(1-\nu_o^2)}{\pi E_o} \bar{F}_{ij} \qquad (4)$$

were E_o is the total deformability modulus of the weak floor strata, ν_o is the corresponding Poisson's ratio, and influence function $\bar{F}_{ij} = \bar{F}_{ij}(B,L,x_{ij})$ where B and L are dimensions of the loaded area, and x_{ij} is the distance between the center of the applied load (j) and the surface point (i) where displacement is considered. The total deformability modulus E_o is related to the elasticity modulus E_o^s and is obtained from triaxial tests: $E_o^s = E_s/(1-\nu_s)$.

Confined Clay Layer: The analysis procedure developed by Taylor and Matyas [6] is based on a solution of Kelvin's equation for a line load acting within an infinite solid. For a constant E_s the immediate surface displacement can be expressed as:

$$F_{ij} = 3 \frac{H_f}{B\pi E_s} \alpha_o \qquad (5)$$

where H_f is the total weak floor strata thickness, and α_o is an influence function [3].

Vlasov's Elastic Layer Model: Considering a finite layer thickness of weak floor strata in the x-y plane (vertical plane), its displacements at any point are given in Eq. 6 (Vlasov [7], Selvadurai [5]):

$$u(x,y)=0, \quad w(x,y) = w(x,o)h(y) \qquad (6)$$

where w(x,o) is the vertical surface displacement, w(x,y) and u(x,y) are the vertical and horizontal displacements at any point, and h(y) is a function describing the variation of displacement w(x,y) in the y direction. Surface subsidence can then be calculated from Eq.6 and from more detailed formulations included in [3,5,7].

Non-Homogeneity in Substrata Properties: An Approximate Method for including non-homogeneity is based on the Gorbunov-Posadov [8] analysis which assumes that the surface settlement of a non-homogeneous elastic halfspace depends strictly on 1) stress distribution according to Boussinesq's solution, and 2) equivalent deformability modulus E_{oe} of the immediate floor strata taking into account the $E_o(y)$ distribution below the foundation center as well as the vertical stress distribution $\eta(\frac{y}{B})$. E_{oe} may be calculated as follows:

$$E_{oe} = \sum^{3B} \eta_i h_i / \sum^{3B} \frac{\eta_i h_i}{E_{oi}} \qquad (7)$$

where η_i and E_{oi} are the mean values of respective variables within the sublayer i of h_i thickness.

Pillar Deformability

It is assumed that pillars can deform along the y-axis (vertical), according to one-dimensional compression. Thus, the coal seam may be represented by a set of linear springs sandwiched between two model elements- floor strata and overburden strata composite beam. Pillar deformations must be subtracted in calculating equivalent beam displacements. The additional pillar deformation component is given by:

$$F^*_{ij} = H_p/BE_p \qquad (8)$$

where E_p is the modulus of elasticity of coal, and H_p is the pillar height. The relative significance of pillar deformations increases as the weak floor strata get stiffer.

Roof and Floor Composite Beam Characteristics

The overburden and immediate floor strata are assumed to be horizontally bedded resting on intact coal (barrier) pillars and on a number of pillars within the panel being mined. The beam is assumed to be simply supported in this model but this will be modified in future. According to Jeffrey and Daemen [9], such a system can be modeled as a composite beam whose behavior depends on the degree of bonding between layers. The overburden loads provide some degree of shear resistance along interfaces. Furthermore since beam deflections and curvature are expected to be small, the different layers may be considered bonded for the purposes of this model. Flexural rigidity of such a composite beam may then be calculated as:

$$EI = \int_Z y^2 E(y) dZ \qquad (9)$$

where Z is the area of the overburden under consideration, E is the modulus of elasticity and I is the moment of inertia of the beam cross-section.

Time-Dependent Model Analysis

Time-dependent deflections of the model composite beams are assumed to be related to rheological properties of the weak floor strata only. Viscoelastic behavior of such strata may be described by a Burger's model, Chugh et al [10], or a Zener or other, simple linear models. Based on laboratory creep studies , Chugh et al [10] found that the immediate floor strata behavior of one mine may be characterized as linear viscoelastic in the range of 40-80% of the failure stress. This is significant since most partial extraction room-and-pillar mines should have stresses within this range (safety factor between 1.2 to 2.5).

Alfrey's analogy permits transformation of an elastic deformation state into a corresponding linearly viscoelastic one. Using this analogy, the time-dependent deformability modulus may be defined as:

$$E_o(t) = E_o(o) \cdot f(t) \qquad (10)$$

where $E_o(o)$ represents time-independent deformability modulus and f(t) reflects stress-strain-time behavior of a rheological model, Biernatowski and Pytel [2], Gatti and Jori [11]. For the Burger's model:

$$f(t) = 1 + \frac{E_2}{E_1}(1 - \exp(-\frac{E_1}{N_1} t)) + \frac{E_2}{N_2} t \qquad (11)$$

where E_1, E_2, N_1, N_2 are the elastic and viscous parameters of the Kelvin-Voigt and Maxwell units respectively (see Fig. 3).

A computer program was developed in Fortran IV language to solve the above model formulation equations. The program was checked for its validity and accuracy for analysis of simple beam problems for which solutions are available.

APPLICATION OF THE MODEL TO A CASE STUDY MINE IN ILLINOIS

Mine Description and Area Geology

The results of Chugh's [12, 13] recent field studies at an Illinois mine were used for model verification. Overburden thickness at the mine averages 80 m with a typical cross-section consisting of 45 m of glacial deposits (clayey soils), 30 m shale, 5 m limestone and 1-2 m shale which forms the immediate roof. The relatively flat coal seam is 1.5-2.0 m thick, mine openings are 6.0 m wide and pillars 15 m (solid) wide in a panel which is approximately 1300 m X 300 m. The immediate floor strata consist of 2 m of water sensitive claystone which is underlain by more competent beds such as mudstone and shale.

Subsidence Monitoring

Surface subsidence has been monitored at this mine over a room-and-pillar mining panel for the past two years; Chugh et al [12]. Two main subsidence monitoring lines (Fig 4) were located over the panel at about 45° and -45° angles with respect to the longitudinal axis of the panel. All surveys were conducted to meet or exceed accuracy standards for Second order Class II surveys.

Measurement of Pillar Settlement Underground

The panel over which surface subsidence was measured was also instrumented underground for monitoring convergence, sag, differential movements, and pillar settlements; Chugh et al. [13]. Pillar settlement was calculated by measuring movements of the immediate roof strata with respect to an assumed fixed point about 6.0 m below the coal seam. A multiple point borehole extensometer (MPBX) was also installed in the roof vertically above the setup in the floor. The distance between the MPBX anchor in the immediate roof and the top of the floor was measured with a tape extensometer having an accuracy of about 0.3 mm. The positions of monitored pillar (point S) underground, mine workings and lines along which surface settlements were measured, are shown in Fig 4.

Deformation Properties of Immediate Roof and Floor Strata and Coal Seam

Deformation properties of overburden material were obtained from an earlier study. Similar properties for coal seam and floor strata were determined by Chugh et al [14]. The deformation properties of immediate floor strata were determined in the field using the ISRM recommended surficial Plate Load Test procedures. In this technique, determined load-deformation relationships reflect the deformation properties of the upper layers to a depth of about two times the size of the plate. The rock cores taken from depths upto about 6m below the coal seam were also studied in the laboratory for time-dependent and time-independent stress-strain behavior and index properties, Chugh et al [10].

Model Application

Overburden lithology with equivalent cross-section representing deformability of each stratum is presented in Fig 5. Flexural rigidity of non-composited, equivalent beam representing overburden strata was calculated according to Eq. 9 as follows:

$$EI = \int_Z y^2 E(y)\,dZ = E_3 \int_Z y^2 E(y)/E_3 \,dx\,dy = E_3 \sum_{y=1}^{3} E_i (h_i d_i^2 + h_i^3/12)/E_3$$

where h_i, E_i are the thickness and elasticity modulus for the i-th layer, d_i is the distance from the center of the i-th layer to its centroid. Finally:

$$EI = 6.89E6[1.0(5.0*13.59^2+5.0^3/12)+0.5(30.0*3.91^2+30.0^3/12)+0.005(45.0*41.41^2+45.0^3/12)] = 1.87E10\ kNm^2,$$

$$GZ = 2E_3F/(1+\nu) = 2*6.89E6*20.225/(1+0.2) = 2.322E8\ kN,$$

where GZ is the shear rigidity of the beam.

Similar data for immediate floor strata are shown in Fig 6. The calculated reactions and displacement profiles for all immediate weak floor strata models are presented in Fig. 7a-b. The value of $E_o = E_{oe} = 7.06E6$ kPa

Sensitivity Analysis

Sensitivity analyses were conducted on several variables given below to determine their relative importance (notations are explained in Fig. 8):

$$\alpha_1 = EI/E_oB^4,\ \alpha_2 = H_f/B,\ \alpha_3 = L/B,\ \alpha_4 = N$$

and $\alpha_5 = E/E_oH_p$.

Based on observed data from the mine, the following average values of α_i were calculated.

$$\alpha_1 = 0.24,\ \alpha_2 = 24,\ \alpha_3 = 0.14,\ \alpha_4 = 15,$$

and $\alpha_5 = 6.6$. Ranges of α_i variability were then established as follows.

$[\alpha_1]=[0.005,\ 1.25]$, $[\alpha_2]=[0.12,\ 0.72]$,

$[\alpha_3]=[1.0,\ 2.0]$,

$[\alpha_4]=[5,\ 25]$, $[\alpha_5]=[2.0,\ 12.0]$.

Additional variables below were introduced to account for non-uniform pillar settlements within a panel.

$$W_1 = \frac{1}{NB} \sum^n Y_i, \quad W_2 = \frac{1}{NB} \left(\sum^n (Y_i - W_1 B)^2 \right)^{1/2}$$

$$R_1 = \frac{1}{NBE_o} \sum^n X_i,$$

$$R_2 = \frac{1}{NBE_o} \left(\sum^n (X_i - R_1 BE_o)^2 \right)^{1/2}$$

where N is the number of pillars in the panel.

Sensitivity analyses (see also Fig. 9a-s) were conducted as follows:
a) One α_i variable was chosen to be independent and the remaining variables α_k were assumed to be equal to their corresponding average values,
b) Y_i/B and X_i/BE_o were considered as the output data where Y_i is the pillar settlement, and X_i is the corresponding reactive force at point i,
c) values of Y_i and X_i were calculated for each of the weak floor strata models for at least five (5) values of α_i, and
d) the calculations were repeated for all α_i variables.

The results of sensitivity analyses are summarized below.
1. Parameter $\alpha_1 = EI/E_o B^4$ has the strongest influence on the system. This parameter represents ability of the system to transfer additional load on the barrier pillars and it is closely related to the relative stiffness of the system. Large overburden stiffness or large weak floor strata deformability lead to excessive barrier load transfer which may result in a significant increase in non-uniformity of reactions as well as lateral and vertical movements.

2. The α_4 parameter is closely related to the beam length (panel width and number of pillars) over weaker foundation. This parameter strongly affects the beam stiffness which is inversely related to the width of the panel. For the range of variability assumed, the weak layer thickness H_f is not very important.

3. Parameter $\alpha_5 = E_p B/H_p E_o$ can be neglected in the analysis, since its effects are secondary. This is because weak floor strata are much more deformable than coal.

4. The constant value of the coefficient of variation $\nu_R = R_2/R_1$ and approximately constant ν_w for any α_i value, indicate that the form of load and settlement distribution remains the same, and are almost independent of α_i.

5. The relative variation of settlements is 10-100 times greater than corresponding relative changes in reactions.

6. An increase in α_3 increases settlements linearly due to an increase in stress on the pillar because of changed mining geometry.

Development of Time-Dependent Behavior and Final Model Verification

Sensitivity analyses indicated that distribution of support reactions along the panel was relatively independent of weak floor deformation properties within a resonable range of E_o variations (see Fig 10a). It can therefore be assumed that the slope (Fig 10c) is a symmetrical influence function, which for the axis parallel to the direction of mining, may be approximated by the following expression.

$$\frac{\partial Y_s}{\partial x} = \frac{C}{A^2 (x_s - x)^2 + 1} \qquad (12)$$

where constants A and C can be derived from slope of the settlement curve and from the limiting condition:

$$Y_s = \int_{-\infty}^{\infty} \frac{\partial Y_s}{\partial x} dx \qquad (13)$$

where Y_s is the total elastic settlement in the center of the panel (Fig 11a), and x is the distance between the present position of mine workings (coordinate x) and the given point (x_s). Assuming the weak floor strata to be a visco-elastic medium without memory, the total pillar settlement after time T may be expressed by the following integral:

$$Y(x,T) = \int_0^T \frac{\partial Y_s}{\partial t} f(T-t)\,dt = \int_0^T \frac{Cv\, f(T-t)\,dt}{A^2 (x_s - vt)^2 + 1} \qquad (14)$$

where v is the constant average rate of mine advance and x is the distance between a given point and the barrier pillar (first mine opening) far enough away to consider the trough as stationary. The function f(T-t) has a form of Eq 11. Constants for the model were evaluated from observed pillar settlement as a function of time and from subsidence surveying results. Using the limiting conditions:

1. $\frac{\partial Y_o}{\partial T}\Big|_{T=t_1} = Y_o'(t_1)$ and $\frac{\partial Y_o}{\partial T}\Big|_{T=t_2} = Y_o'(t_2)$

where t_1, t_2 are arbitrarily chosen values of time and $Y_o'(t_1)$ and $Y_o'(t_2)$ are corresponding values of slope obtained from observed settlements.

2. $Y(x_s, T_1) = S_o(T_1)$ is the total value of the observed surface subsidence (at the point immediate above the observed pillar) in the field after time T_1,

3. $E_2 = E_o$ represents the elastic solution.

The following relationships and model parameters were obtained (for v=1m/day and X_s = 900m).

$$\frac{\partial Y_s}{\partial x} = 1.198E\text{-}4/(4.9307E\text{-}4(x_s - x)^2 + 1)$$

and E_1=6.923E8 kPa, E_2=7.060E6 kPa,

N_1=7.692E9 kPa day,

N_2=2/651E9 kPa day.

Calculated pillar settlement and time-dependent surface subsidence are compared with observed values in Fig 11a-b. The model predicted parameters were compared with those calculated by Chugh [10] from floor heave observations in the field. After a long period of time the settlement slope should be independent of the Kelvin-Voigt's unit (short-term influence) and rate of advance (v) for mine workings. For such a case, $dY(x,T)/dT = Y_s(x)\, E_2/N_2$ = 4.26E-5 m/day.

Initial (elastic) magnitude of floor heave H_s may be assumed proportional to pillar elastic settlement: $H_s(x) = Y_s(x)\beta$ where β is the ratio of floor heave to pillar settlement after time T and it may be calculated as:

$$\frac{H(x,T)}{Y(x,T)} = \frac{\beta\, Y_s(x)}{Y_s(x)} \frac{(1+D^*T)}{(1+DT)} = \beta \qquad (15)$$

where $D^* = E_2^*/N_2^*$ (parameters from [10]) and $D = E_2/N_2$ (parameters obtained from analysis above). The value of D^* must be equal to D if the numerical model presented here is valid.

*Considering west side of the mine,
E_2^* = (4.25 to 1.24)E5 kPa

N_2^* = (4.49 to 2.40)E12 kPa sec.
D^* = (10.54 to 1.64)E -3 per day
D =(2.66)E-3 per day.

Thus there is a general agreement between the model predicted and field observed values.

The constant β was also determined from field floor heave rate observations, Chugh et al [13]. Approximate average slope was calculated as dH(T)/dT = 2.54E-2 m/day.

After 400 days ($v=\infty$), the value of the field-observed floor heave is approximately equal to H(400) = 2.54E-2* 400= 0.1016 m. The value of β is given by: $H(400)/Y_s(1+400D)$= 3.08 which is again in agreement with field observations (β= 3.22), Chugh et al [13].

It should be pointed out that the model calculated parameters E1,E2,N1,N2 may not have the same values as estimated from floor heave data observed in the field. This is because model values represent the composite for all layers of the immediate floor strata considered in the model.

CONCLUDING REMARKS

Results presented above have confirmed the usefulness and relative accuracy of the model. The critical point of analysis is the input data file, which includes data concerned with either time-independent conditions (opening and pillar geometry, lithology, geologic conditions etc.) or external influences which change with time (mining geometry, mining rate, deformability of weak strata etc.).

This approximate two-dimensional solution of the overburden-pillar-floor interaction problem has made it possible to assess relative importance of different parameters governing the system.

The model has typical advantages and limitations of any approximate approach. Since it is a relatively fast computational technique, it requires regular panel geometry and mining sequence. Three dimensional problems such as an intersection may also be solved. For such a problem, a beam may be replaced by an elastic plate resting on blocks representing coal pillars. Techniques of solution remain the same, but much more computer memory is required.

Even though the results of the numerical model are encouraging, these results should be treated as preliminary since they are based on only one case study. Additional similar studies must be conducted to develop confidence in the model and calculation procedures.

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REFERENCES

[1] Krol W., (1964), Statics of Reinforced Foundations with Including Superstructure Rigidity, (in Polish), Arkady, Warsaw (there exists German Translation).

[2] Biernatowski K., Pytel W., (1981) Design model of soil-structure interaction including reliability theory, Proc. X ICSM&FE, Stockholm, v. 2, pp. 45-48.

[3] Pytel, W. (1988), Development of an Analytical Model for Solving Roof-Coal Pillar-Floor Interaction Problems, Internal Research Report, Department of Mining Engineering, Southern Illinois University at Carbondale.

[4] Zemochkin B. N., Sinitsyn A.P., (1947), Practical method for calculation of beams and plates on elastic foundation, (in Russian), Strojizdat, Moscow.

[5] Selvadurai, A.P.S., (1979), Elastic Analysis of Soil-Foundation Interaction, Elsevier Scientific Publishing Company, Amsterdam-Oxford-New York.

[6] Taylor B. B., Matyas E.L., (1983), Settlement of a strip footing on a confined clay layer, Can. Geot. J., v. 20, pp. 535-542.

[7] Vlasov V. Z., Leontiev U. N., (1966), Beams, plates and shells on elastic foundation, Israel Program for Scientific Translations, Jerusalem.

[8] Gorbunov-Posadov M. I., (1953), Analysis of structures on elastic foundation, (in Russian) Moscow, Goss. Izd. Lit. Stroit. Arch.

[9] Jeffrey R. G., Daemen J. J. K., (1984), Analysis of rockbolt reinforcement of layered rock, Proc. Int. Symp. on Rock Bolting, Abisko, ed. O. Stephansson, pp. 173-185. A.A. Balkema, Rotterdam/Boston.

[10] Chugh Y. P., Nath, R., Shankar, S., (1987), Time Dependent Behavior of Immediate Weak Floor Strata from an Illinois Coal Mine, Proc. 6th Int. Conf. on Ground Control in Mining, Morgantown, WV, pp. 204-218.

[11] Gatti G., Jori I., (1981), The creep effect in the soil-foundation interaction, Proc. X ICSM&FE, Stockholm, vol. 2. pp. 115-118.

[12] Chugh Y. P., Kuscu, S., Atri, A., and Sweigard, R. (1988), Subsidence recovery at a shallow partial extraction on room-and-pillar mine in Midwestern United States Seventh Int. Congress on Mine Surveying, June 28-July 2, 1988, Leningrad, USSR.

[13] Chugh Y. P., Chandrashekhar, K., Caudle, R. D., (1988), A Field Geotechnical Study of the Effects of Weak Floor Strata on An Underground Coal Mine in Illinois, 29th U.S. Symposium on Rock Mechanics, June 10-15, Univ. of Minnesota, Minn.

[14] Chugh, Y. P., K. Chandrashekhar, R. D. Caudle (1987), Effect of Soft Floor Interaction on Room-and-Pillar Mining, Annual Progress Report, Mine Systems Design and Ground Control Generic Center Virginia Polytechnic Institute.

a)

b)

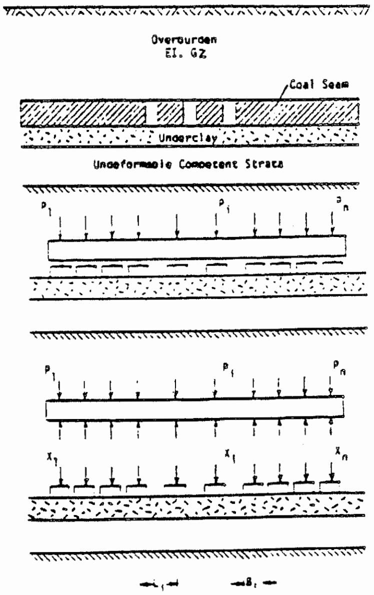


Fig 1 A Schematic of the Roof-Pillar-Floor Interaction Model

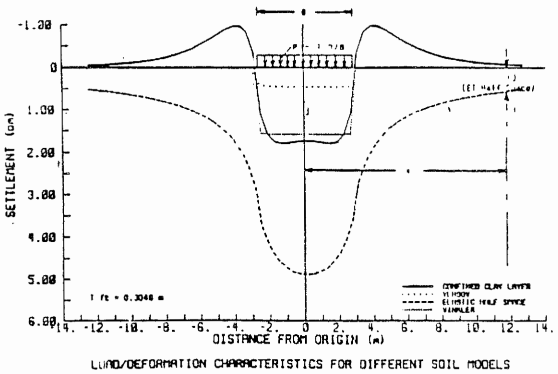


Fig 2 Calculation for F_{ij}

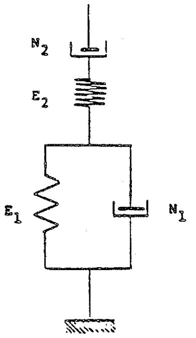


Fig 3 Burger's Model of Visco-Elastic Medium

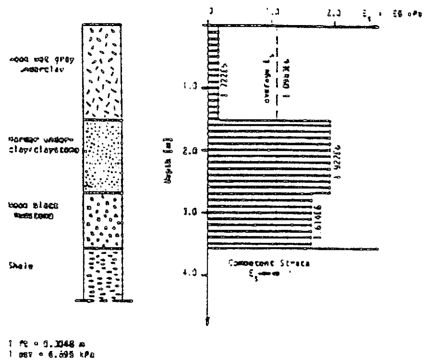


Fig 6 Lithologic Log of Immediate Floor Strata and E_0 Values for Different Layers

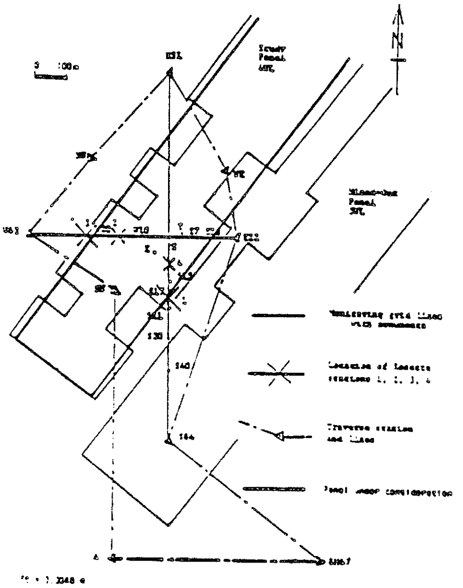


Fig 4 Location of Mine Workings, Monitoring Sites, and Subsidence Monitoring Lines (Chugh et al, [12])

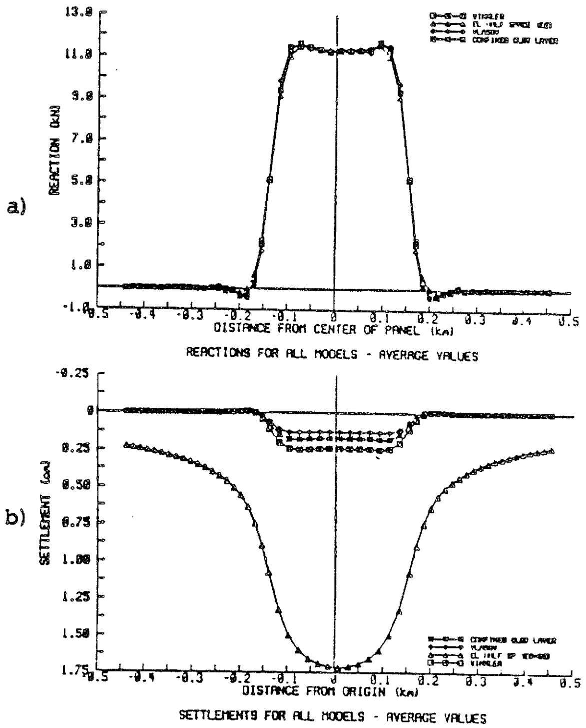


Fig 7 Reaction and Settlement Data for Different Models of Weak Floor Strata

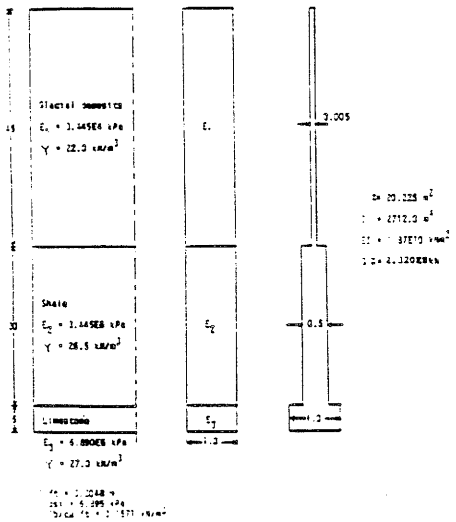


Fig 5 Data for Overburden Lithology and Equivalent Composite Beam

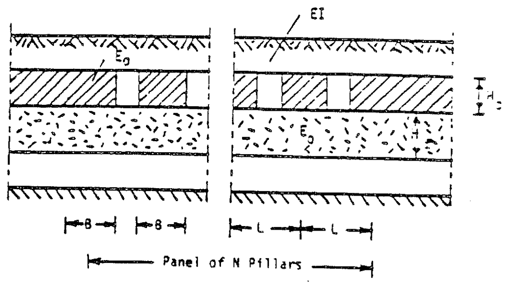
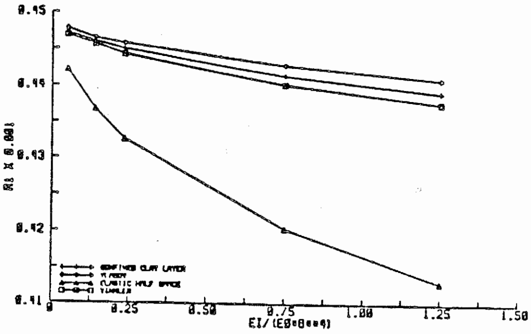


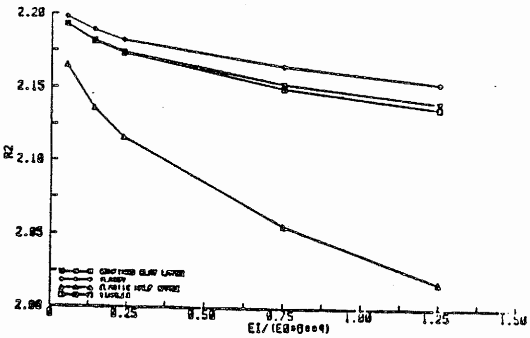
Fig 8 Scheme for Sensitivity Analysis

a)



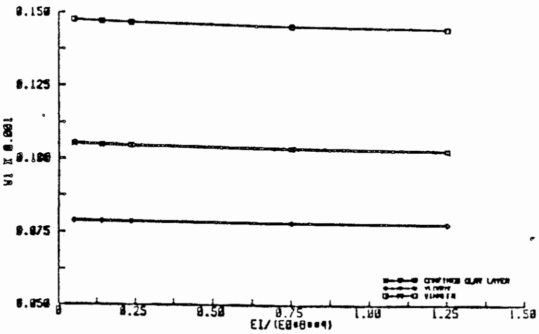
VARIATION OF α1 AND R1 PARAMETERS FOR DIFFERENT SOIL MODELS

b)



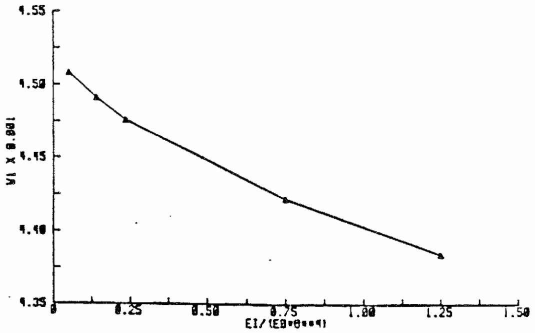
VARIATION OF α1 AND R2 PARAMETERS FOR DIFFERENT SOIL MODELS

c)



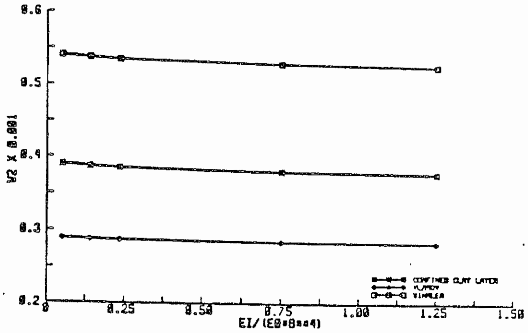
VARIATION OF α1 AND V1 PARAMETERS FOR DIFFERENT SOIL MODELS

d)



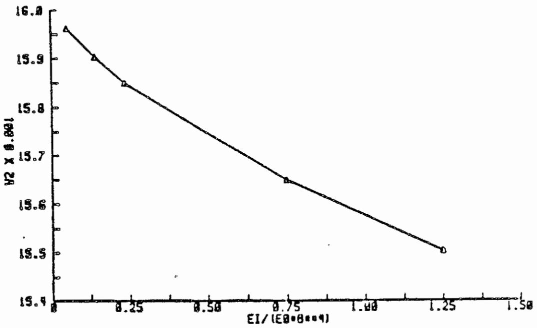
VARIATION OF α1 AND V1 PARAMETERS FOR ELASTIC HALF SPACE MODEL

e)



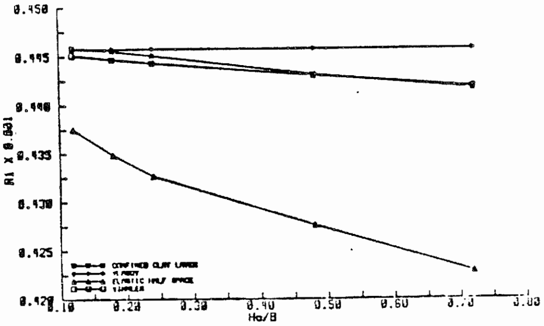
VARIATION OF α1 AND V2 PARAMETERS FOR DIFFERENT SOIL MODELS

f)



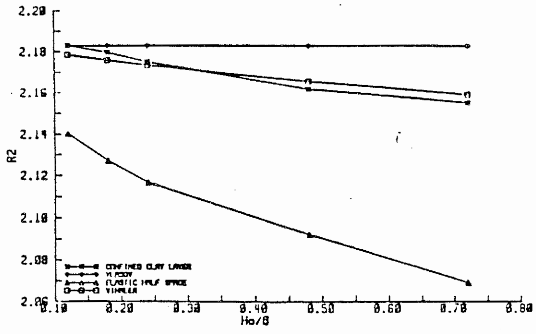
VARIATION OF α1 AND V2 PARAMETERS FOR ELASTIC HALF SPACE MODEL

g)

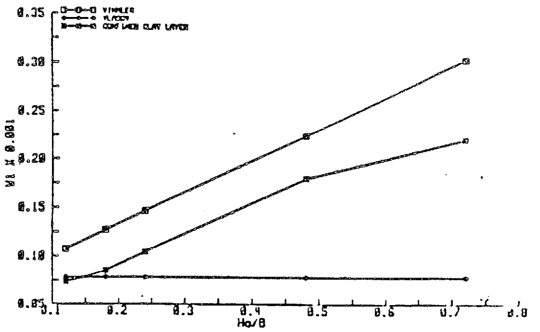


VARIATION OF α2 AND R1 PARAMETERS FOR DIFFERENT SOIL MODELS

h)

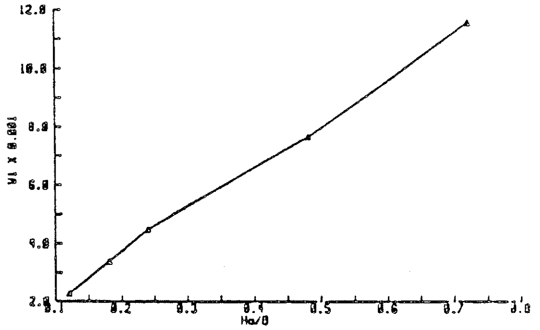


VARIATION OF α2 AND R2 PARAMETERS FOR DIFFERENT SOIL MODELS i)



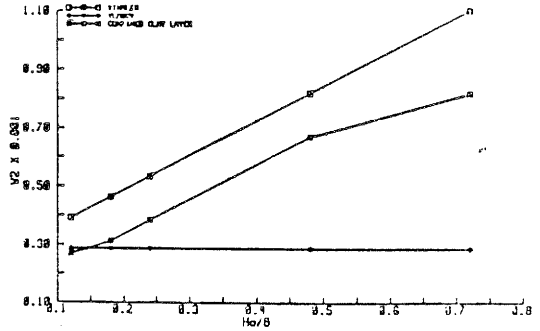
VARIATION OF α2 AND V1 PARAMETERS FOR DIFFERENT SOIL MODELS

j)



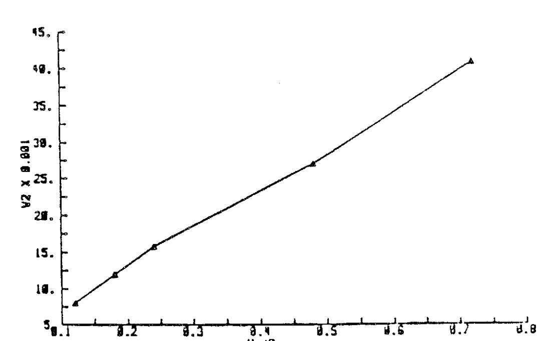
VARIATION OF α2 AND V1 PARAMETERS FOR ELASTIC HALF SPACE MODEL

k)



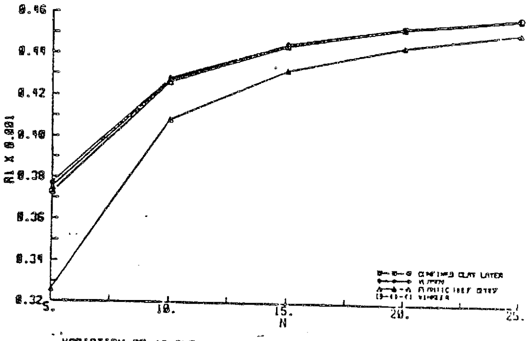
VARIATION OF α2 AND V2 PARAMETERS FOR DIFFERENT SOIL MODELS

l)



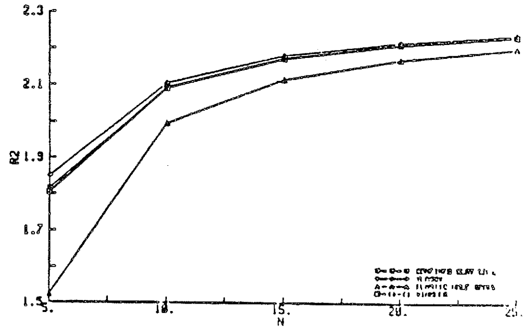
VARIATION OF α2 AND V2 PARAMETERS FOR ELASTIC HALF SPACE MODEL

m)



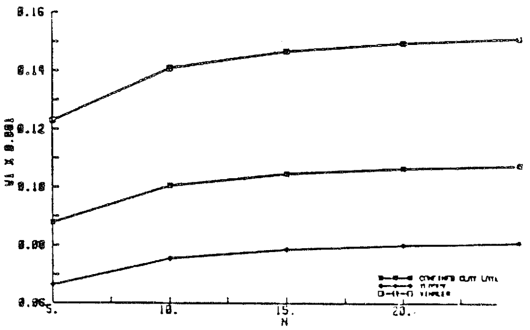
VARIATION OF α4 AND R1 PARAMETERS FOR DIFFERENT SOIL MODELS

n)



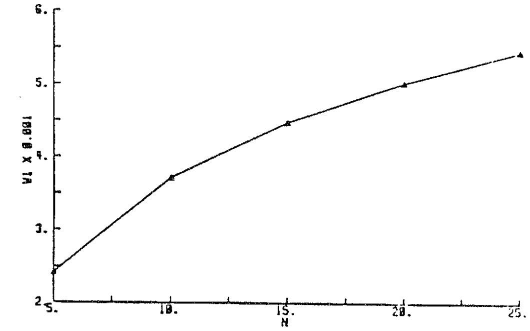
VARIATION OF α4 AND R2 PARAMETERS FOR DIFFERENT SOIL MODELS

o)



VARIATION OF α4 AND V1 PARAMETERS FOR DIFFERENT SOIL MODELS

p)



VARIATION OF α4 AND V1 PARAMETERS FOR ELASTIC HALF SPACE MODEL

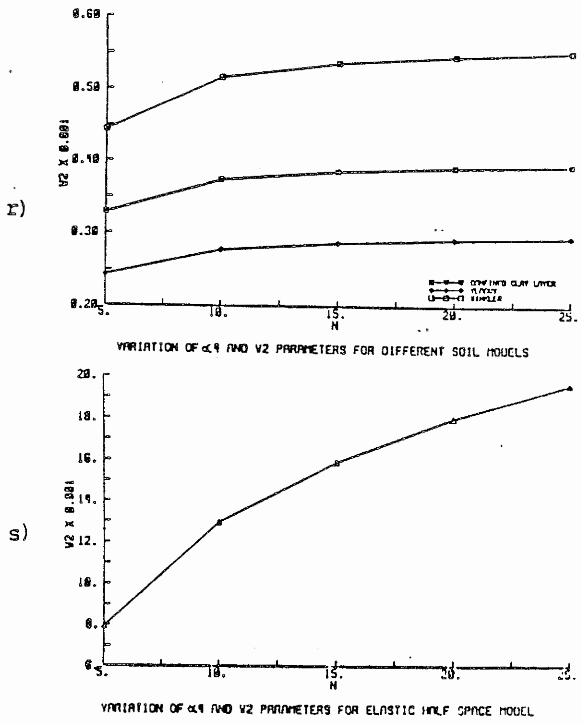
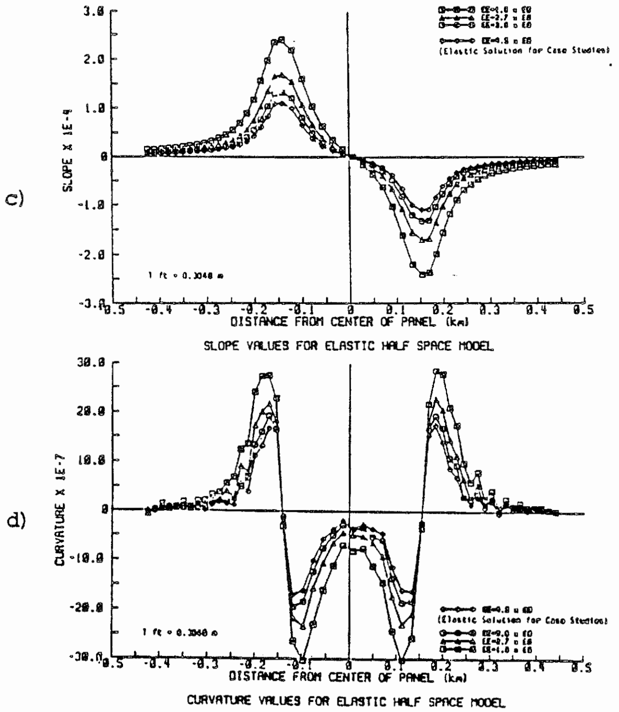


Fig 9 Selected Results of Sensitivity Analyses



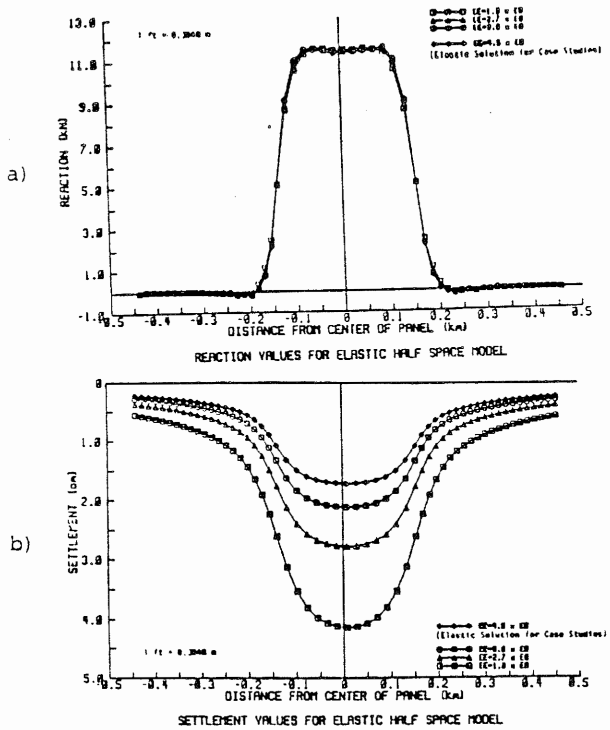


Fig 10 Support Reactions (a), Pillar Settlements (b) Slope (c) and Curvature (d) For Selected Values of E_o (panel of 15 pillars)

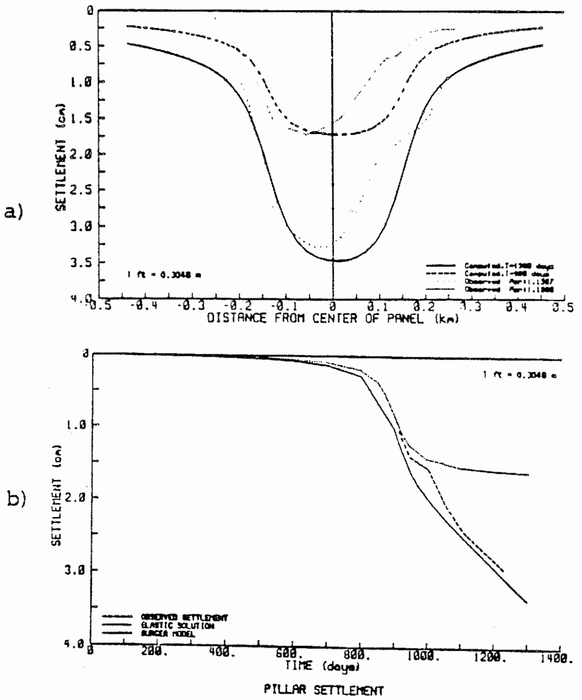


Fig 11 Time-Dependent Surface Subsidence and Pillar Settlement

An Analysis of Roof-Pillar-Weak Floor Interaction in Partial Extraction Room-and-Pillar Mining

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ABSTRACT: The applicability of the beam theory in analysis of roof-pillar-weak floor interaction in partial extraction room-and-pillar mining is presented. The mine structure is modeled as an equivalent multi-indeterminate overburden elastic beam supported by elasto-plastic pillars resting on a viscoelastic layer of immediate weak floor strata underlain by a competent rock mass. The developed analytical model was initially utilized to conduct sensitivity analyses of different variables affecting the mining system, such as the deformability of coal and weak floor strata, thickness of weak floor strata, number of pillars in a panel, width of pillars, width of panel etc. These analyses were then extended to three overburden strata - coal pillar - floor strata lithologies typical of active coal mining areas in Illinois.

1. INTRODUCTION

The rational design of any mining system requires knowledge of the actual load and displacement characteristics in a panel as well in its adjoining areas. This implies a fundamental understanding of roof-coal seam-floor strata interaction, and load transfer within the different parts of the mine due to the mining sequence and resultant surface and subsurface movements associated with mining. To study these interactions, the authors developed an approximate two-dimensional time-dependent analysis technique based on the theory of beams on inelastic foundations (Pytel, et al., 1988). The model was developed with the specific objectives to predict: 1) pillar settlements and associated surface subsidence movements as a function of time due to mining of one or more panels, and 2) transfer of load as a function of time to adjoining pillars or adjoining areas due to pillar settlement of weak floor strata or yielding of pillars. The model can consider different size pillars in a panel, different rates of advance and time lag in mining in different parts of a panel, and up to 50 pillars across a panel. Based on model validation results to date, the authors think the model has significant potential in analyzing the relative magnitude of roof-pillar-floor interaction effects in different geologic settings.

2. MODEL DESCRIPTION

Figure 1 depicts the physical problem involving overburden strata, coal pillars, and floor strata and its idealization as a structural mechanics problem. The two dimensional plane strain model described here is based on the theory of beams on inelastic foundations. A summary of the theoretical background for the model was presented in an earlier paper (Pytel et al., 1988). A more detailed theoretical discussion of the model is currently under preparation by the authors.

In the solution approach, the mine structure is modeled as an equivalent multi-indeterminate overburden elastic beam supported by elasto-plastic pillars resting on a viscoelastic layer of immediate weak floor strata underlain by a competent rock mass. Stratified overburden associated with a coal seam is transformed into a composite beam with stepwise varying flexural and shear stiffness. Overburden strata behavior depends on the degree of bonding between layers, and two extreme cases are considered: 1) the different layers are fully bonded and the overburden acts as a single thick beam, and 2) the overburden strata interfaces are smooth and act as a number of sub-beams.

The overburden load is transmitted to the weak floor strata through segmented rectangular foundations representing coal pillars. Contact stresses at the soil/rock beam interface, which constitute the unknowns in the problem, are approximated by rectangular areas of uniform stress. This is transformed into an equivalent concentrated force acting at the center of the plan area of each element. A coal pillar is represented by a set of nonlinear springs sandwiched between the upper overburden strata beam and the lower deformable weak strata. The non-linear response of the coal pillar is based on studies by Wilson and Ashwin (1972) and Hardy, Christiansen and Crouch (1977). These models imply that coal pillars with width to height ratios greater than six would have a central elastic core of infinite strength. The residual strength behavior of the coal pillar may be characterized as elastic-perfectly plastic or elastic-plastic with strain softening.

The stress-settlement behavior of the weak floor strata is modeled through linear Winkler's model which has been found more appropriate in foundation engineering where beams or plates are resting on relatively thin deformable strata. Time-dependent behavior of the weak floor strata is idealized as a Burger's model in the analysis based on previous studies (Chugh et al, 1987).

The problem solution which consists of the vector of reactive uniformly distributed pressure underneath pillars is obtained using the Maxwell-Betti reciprocal theorem from a system of algebraic linear equations with two global static equilibrium conditions. The horizontal pressure is neglected in the development of this initial model. The model is based on the small strain theory which requires that: 1) the overburden strata thickness should be small as compared to the width of panels including barrier pillars, and 2) the ratio of expected surface subsidence to overburden thickness should also be small. The model output data include the average stress in all pillars, load transfer to adjacent and barrier pillars as a function of time, pillar settlements and surface subsidence.

Application of the model at a mine in central Illinois indicated a significant correlation between the model predicted and the field observed values of pillar settlement and surface subsidence, Figure 2.

The applicability of the model was also evaluated at a mine in southern Illinois where pillar splitting was being considered. The objective of the evaluation was to predict expected surface subsidence movements over a panel due to pillar splitting. No surface subsidence monitoring was performed, and the model evaluation was based on measured and predicted roof to floor convergence in the mine. The deformation properties of floor strata required for analysis were determined from plate loading tests, while similar properties for roof strata and coal were determined from laboratory tests. The determined material constants were not scaled down further. A summary of the results is presented elsewhere, Chugh, and Pytel (1988).

3. SENSITIVITY ANALYSES

After the initial model validation, the relative importance of the different variables affecting the roof-pillar-floor interaction such as the deformability or modulus of coal and weak floor strata (Eo = 63-568 MPa with average value equal to 189.4 MPa), thickness of weak floor strata (Ho = 0.61-5.49 m with average value equal to 1.83 m), and its variability, number of pillars in a panel, width of pillars, and width of a panel were evaluated using the developed model for the study mine in central Illinois. The geometry of a typical panel and the average values for different variables are summarized in Figure 3 and Table 1. These analyses were conducted for three (3) overburden strata-coal pillar-floor strata lithologies (Figure 3 and Table 1). Case 1 represents a large ratio of unconsolidated to consolidated overburden; case 2 indicates the presence of a high strength layer (limestone or sandstone) overlying in the vicinity of the coal seam, and case 3 represents the case for the weak immediate roof. For each of these cases, analyses were conducted for bonded and unbonded overburden strata layers. The overburden strata characteristics of the central Illinois mine are represented by case 3. In addition, analyses representing extremely stiff overburden (Case 4), and extremely low stiffness overburden (Case 5) were also conducted. The results were analyzed for magnitude of pillar settlements and relative changes in pillar pressures for first barrier pillar (No. 30, Figure 3), first panel pillar (No. 32), and central panel pillar (No. 45). The following discussion summarizes results.

Table 1. Typical overburden-coal-floor strata lithologies.

Case	Properties	Layer No. (Figure 3)									
		I	II	III	IV	V	VI	VII	VIII	IX	X
1*, 1a(*)	Thickness [m]	6.09	9.14	9.14	25.91	37.18	3.05	0.91	1.52	1.83	1.83
	Modulus of elasticity E [MPa]	68.9	103.4	103.4	861.8	1379.0	4137.0	1034.2	1034.2	182.9	6895.0
2*, 2a(*)	Thickness [m]	6.09	9.14	9.14	25.91	37.18	3.05	0.91	1.52	1.83	1.83
	Modulus of elasticity E [MPa]	68.9	861.8	861.8	861.8	1379.0	4137.0	1034.2	1034.2	182.9	6895.0
3*, 3a(*)	Thickness [m]	6.09	9.14	9.14	25.91	37.18	3.05	0.91	1.52	1.83	1.83
	Modulus of elasticity E [MPa]	68.9	861.8	861.8	861.8	1379.0	1034.2	1034.2	1034.2	182.9	6895.0

* - bonded layer interfaces

(*) - unbonded layer interfaces

3.1 Effect of overburden stiffness (Figure 4)

1. Pillar settlement and pillar loads generally decrease with increasing overburden stiffness for a given panel layout.

2. Decreasing overburden stiffness may transform a subcritical panel into a critical or supercritical panel. For overburden stiffness values typically encountered in Illinois, lower (≈ 8 pct) pillar loads and pillar settlements may develop in the central portion of the panel. Similar values at the edges of the panel are, however, increased due to load transfer towards the barrier pillar.

3. Large overburden stiffness may significantly increase the angle of draw value and uplifting of the ground over the unmined areas. For typical overburden lithologies in active mining areas in Illinois, the angle of draw values vary 18 deg. to 29 deg. (for zero movement), if ground uplift is neglected. The maximum value of the estimated ground uplift is 0.7 mm which is within the accuracy limits of second order-class II surveying. The predicted values of the angle of draw are similar to those observed (17-35 deg. based on 3 mm of movement in the Illinois Basin mines.

4. For cases 1-3 overburden lithologies, the predicted inflection points are located 20-25 m away from the panel barrier towards the center of the mined-out panel. This is consistent with subsidence observations at the central Illinois mine, where inflection points were located in the mined-out panel and distances from the edge of the panel barrier varied 6.7 m to 47.2 m, with an average value of 27.3 m.

5. Slippage between the different strata in the overburden (unbonded case) can increase the slope and curvature values by 60 pct and by 200 pct as compared to the bonded layers case.

6. Roof falls, based on slope and curvature, are expected to occur adjacent to the panel barrier pillars and up to a distance equal to about 10 pct of the panel width.

7. Slope and curvature values of the pillar settlement/subsidence profile are significantly more sensitive to the degree of bonding between layered strata than to the changes in overburden stiffness typically encountered.

3.2 Effect of weak floor strata thickness (Figure 5)

1. Increasing thickness for a given overburden stiffness increases pillar settlement linearly, as would be intuitively expected. The effect of overburden stiffness (unbonded case) on pillar settlements across the panel may be neglected. For the bonded case, however, the settlements of the first panel pillar are significantly smaller (20 pct).

2. The changes in pillar pressure on the first barrier pillar and central panel pillar due to thickness of weak floor strata are less than 5 pct and may be neglected. The effect of overburden stiffness (unbonded cases) on changes in the pillar pressure due to thickness of weak floor strata are also very small.

3. Overburden stiffness, for both unbonded and bonded cases, is significant only for settlements and pillar pressures of the first panel pillar.

3.3 Variability of weak floor strata thickness (Figure 6)

The value of weak floor strata thickness was varied across the panel between 0.61 to 1.83 m, according to the following equation: $H_o(x) = 1.219 [1 + 0.5 \sin (a \pi \cdot x/Wp)]$ where Wp = 13.72 m and Wp represents pillar width, and "x" is the distance in meters from the outermost pillar (No. 1, Figure 3) and "a" is the period of variation. Figure 6 summarize the results obtained and indicate that the local variability in thickness of weak floor strata may cause an unsymmetric subsidence profile, change observed values of the angle of draw, locally decrease or increase surface subsidence and slope and curvature of the subsidence profile. Determination of the variability of weak floor strata thickness is therefore considered very important before designing a mining system. Abrupt changes in slope and curvature values may lead to roof failures in unexpected areas.

3.4 Deformability of immediate floor strata (Figure 7)

1. More and more load is transferred to the barrier pillars with decreasing modulus of deformation. Pillars within the panel carry less load. The pillar settlement and the slope and curvature of the subsidence profile are, however, significantly increased, which may increase the likelihood of roof failures adjacent to the panel barrier. Structural instability is therefore likely due to roof failures rather than due to pillar loads.
2. A low deformation modulus of weak floor strata tends to increase the angle of draw as well as uplifting of the ground.
3. The inflection point of the profile tends to shift toward the center of the panel with decreasing modulus of deformation. A maximum shift of about 5 m is noted for the three cases analyzed.
4. The curvature of the profile across the panel, in the presence of weak floor strata, are significantly higher than where no weak floor strata are involved. This may lead to roof instability problems all across the panel.

3.5 Load transfer due to time-dependent behavior of immediate floor strata

Additional pressure distribution on the pillars due to time-dependent deformation of immediate floor strata is depicted in Figure 8. Viscoelastic parameters utilized in the analysis were E_1=1.723 E7 kPa, E_2=1.723 E5 kPa, N_1=1.914 E8 kPa day, N_2=6.470 E7 kPa day. The results indicate that additional pressure increases with time in the center of the panel and on the barrier pillar, while it decreases on the edges of the panel for unbonded overburden strata. The reverse may occur for bonded strata. Additional load transfer due to time-dependent behavior of weak floor strata does not seem to be critical from a pillar stability point of view unless pillar settlements become very large. The load transfer towards the panel barriers occurs very slowly for the cases analyzed. The inflection points appear to remain unchanged, but the angle of draw tends to increase slightly.

3.6 Influence of pillar yielding

Pillar yielding primarily affects deformation of the mining structure with settlement, slope, and curvature increasing by 50, 20 and 15 percent respectively. Additional pillar pressures due to pillar yielding are insignificant for extraction ratios encountered in partial extraction room-and-pillar mining and for the cases analyzed. Angle of draw is relatively unchanged, but the point of inflection tends to shift toward the panel barrier.

3.7 Influence of panel width

For panel widths analyzed (6-45 pillars, Wp = 13.7 m), pillar loads and settlements at a point in the panel appear to be independent of the panel width, both for unbonded and bonded overburden strata.

4 CONCLUDING REMARKS

This paper has analyzed the relative effects of important variables affecting the roof-pillar-floor interaction in partial extraction room-and-pillar mining systems typically practiced in Illinois basin coal mines. The analyses are based on a simplified two dimensional analytical model utilizing the theory of beams on elastic or inelastic foundations. We think that the structural idealization approach and the analytical model have significant potential for 1) practical mine design, 2) ability to analyze complex geometries which are mined at different times, 3) ability to incorporate non-linear time-independent behavior of coal measure rocks, 4) possibility of considering statistical variation of system parameters in estimating reliability for the entire panel. The developed analytical model is one of the few available models which can be easily applied for routine mine design and permits determination of surface subsidence and average load acting on pillars. An extension of the model to analyze three-dimensional geometries is a logical step and funding is presently being sought to undertake the development.

ACKNOWLEDGEMENTS

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REFERENCES

Chugh Y. P., Nath R., Shankar S., 1987. Time dependent behavior of immediate weak floor strata from an Illinois coal mine. Proc. 6th Int. Conf. on Ground Control in Mining, Morgantown WV, 204-218.

Chugh Y.P., Pytel W.M., 1988. A technical feasibility study of pillar splitting at Eagle 2 mine. Report submitted to Peabody Coal Company. July.

Hardy M. P., Christiansen M., Crouch S. L., 1977. Hybrid computer system for optimization of extraction procedures in tabular coal deposits. Dept. of Civil and Min. Eng. Univ. of Minnesota, Minneapolis, Minnesota 55455, Final report, USBM Contract Report.

Pytel W. M., Chugh Y. P., Zabel B., Caudle R. D., 1988. A simplified two-dimensional analysis of the roof-pillar-floor interaction problem in coal mines. Proc. 7th Int. Conf. on Ground Control in Mining (ed. Syd S. Peng), Morgantown 1988, 271-281.

Wilson, A. and Ashwin D. P., 1972. Research into the determination of pillar size. Part I. A hypothesis concerning pillar stability. The Mining Engineer, 131: 409-417.

Figures:

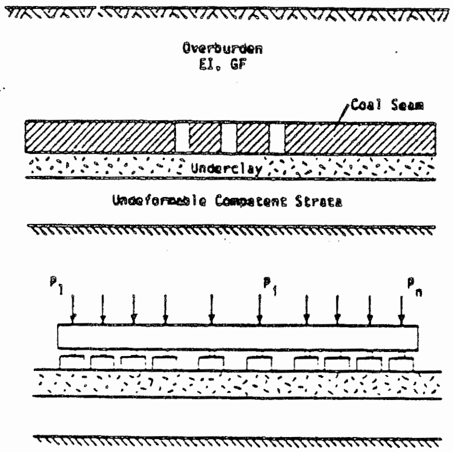


Figure 1. Mining structure and statically equivalent system.

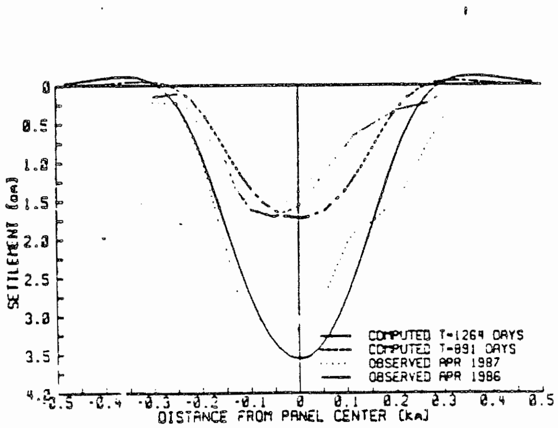


Figure 2. Model predicted and mine observed values of surface subsidence (weaker correlation on the right hand side is because panels in the areas were developed at a later time than those on the left side).

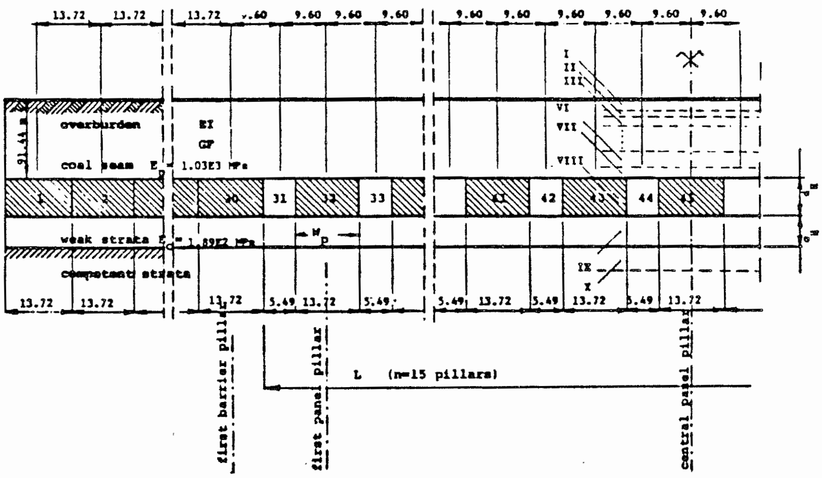


Figure 3. Average mining situation.

a) b)

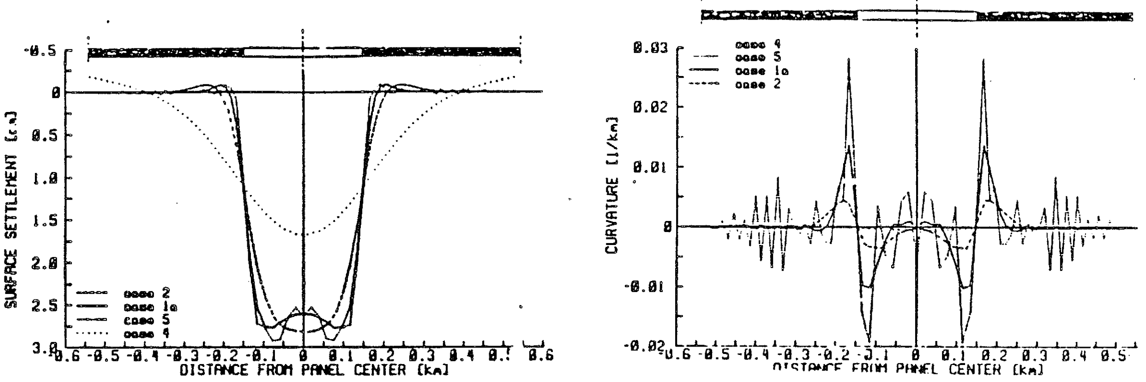


Figure 4. Influence of overburden stiffness on surface settlement (a) and curvature (b). Case 4 (EI=2.87E11 kNm^2), case 5 (EI=2.87E7 kNm^2).

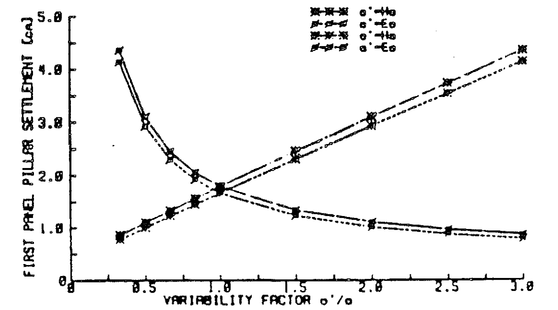


Figure 5. Selected results of sensitivity analysis for case 1a (—) and 2a (--).

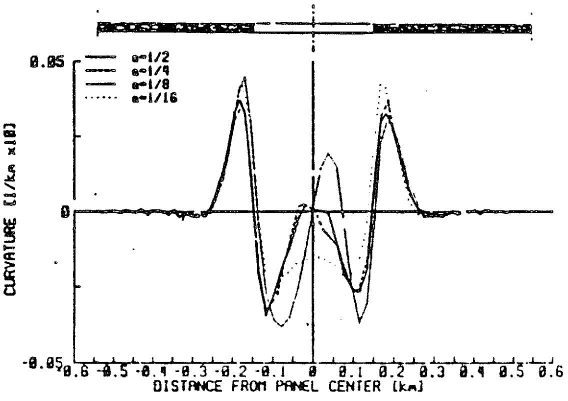


Figure 6. Influence of weak floor strata thickness local variability on surface curvature (case 1).

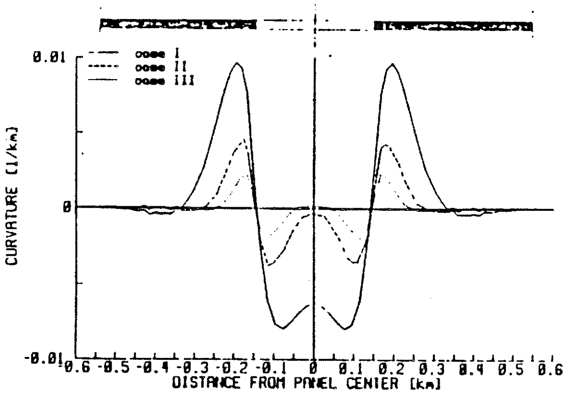


Figure 7. Influence of immediate floor strata deformability on surface curvature.

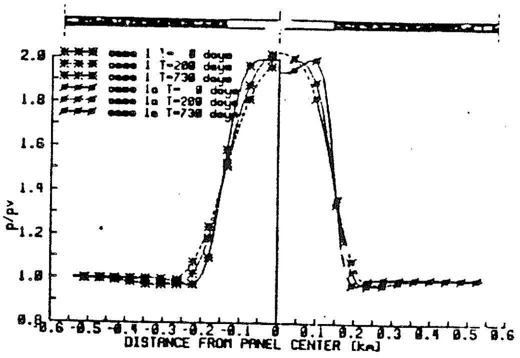


Figure 8. Time dependent pressure transfer for case 1 (left) and 1a (right), p-pillar pressure after mining, p_v-before mining.

A P P E N D I X II

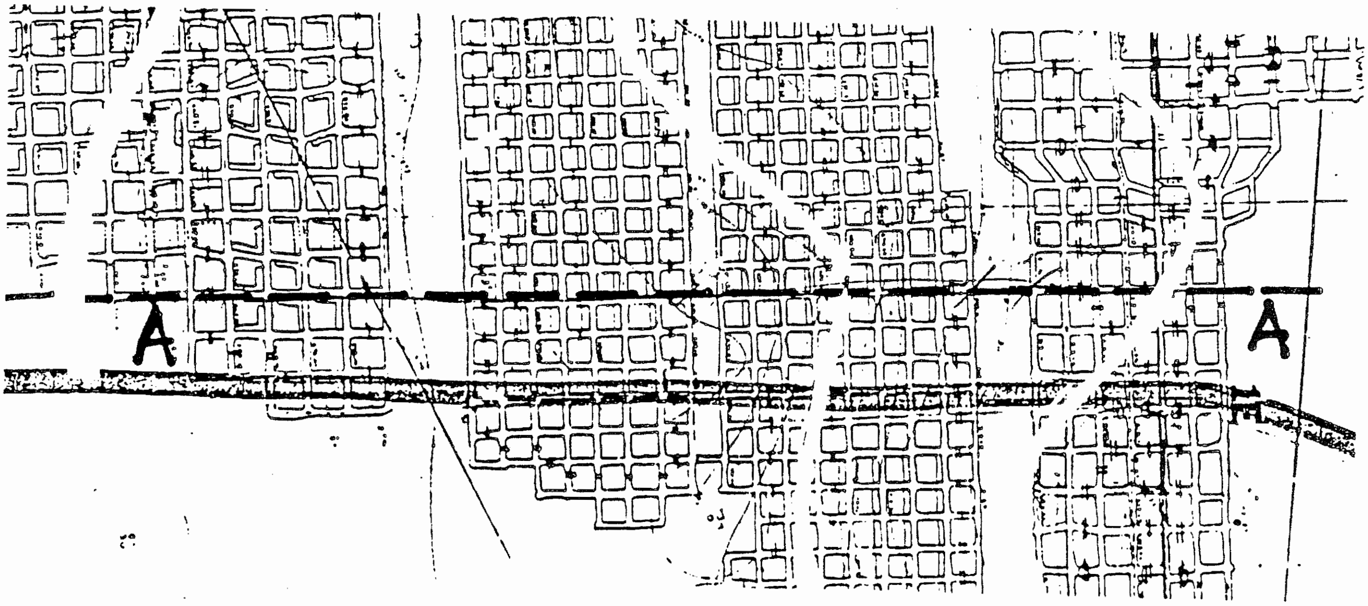
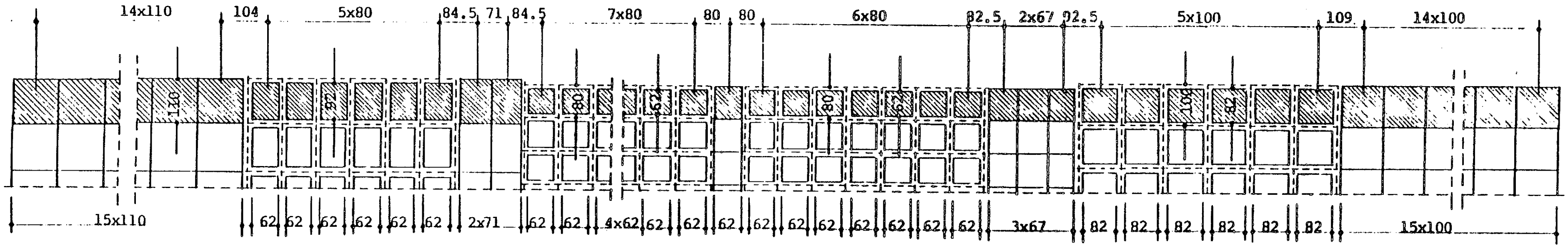


Figure II-1 Underground Mine Layout For Example Problem.



PANEL CALCULATION SCHEME

dimensions in [ft]

Overburden Thickness H_o = 379 ft
Coal Seam Thickness H_p = 86 in
Weak Floor Strata Thickness H_c = 2.5 ft
Opening Widths W= 20 ft
Overburden Flexural Stiffness EI= 7.273E14 psi
Overburden Shear Stiffness GF= 2.751E08 lb/in
Modulus of Weak Floor Strata Deformability E_c = 4.02E4 psi
Weak Floor Strata Poisson's Ratio = 0.35
Deformation Modulus for Coal E_p = 2.0E5 psi

Figure II-2 Panel Layout Along Section A-A for Analysis of Subsidence
(Example Problem)

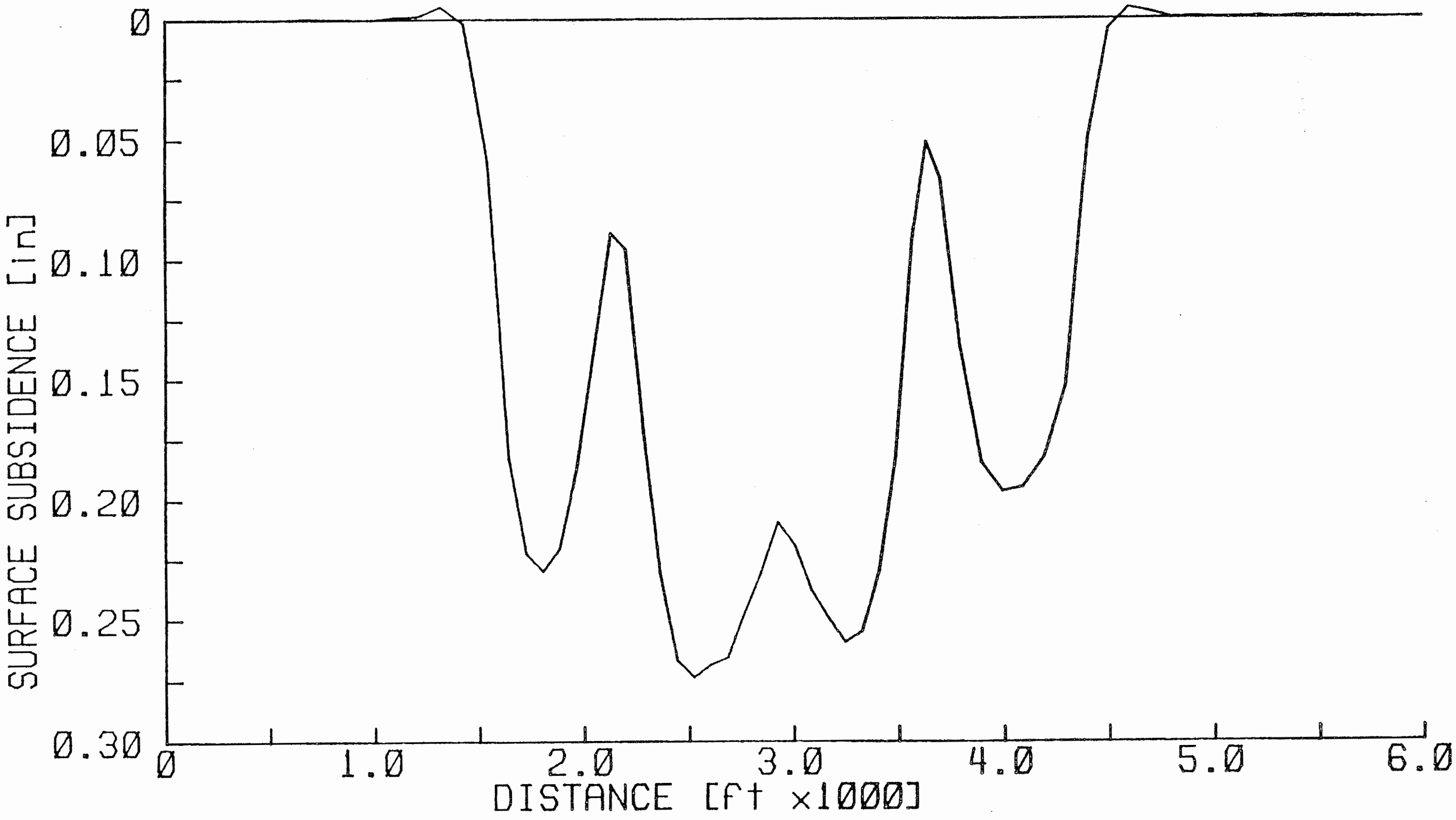


Figure II-3 Surface Subsidence Along Section A-A.

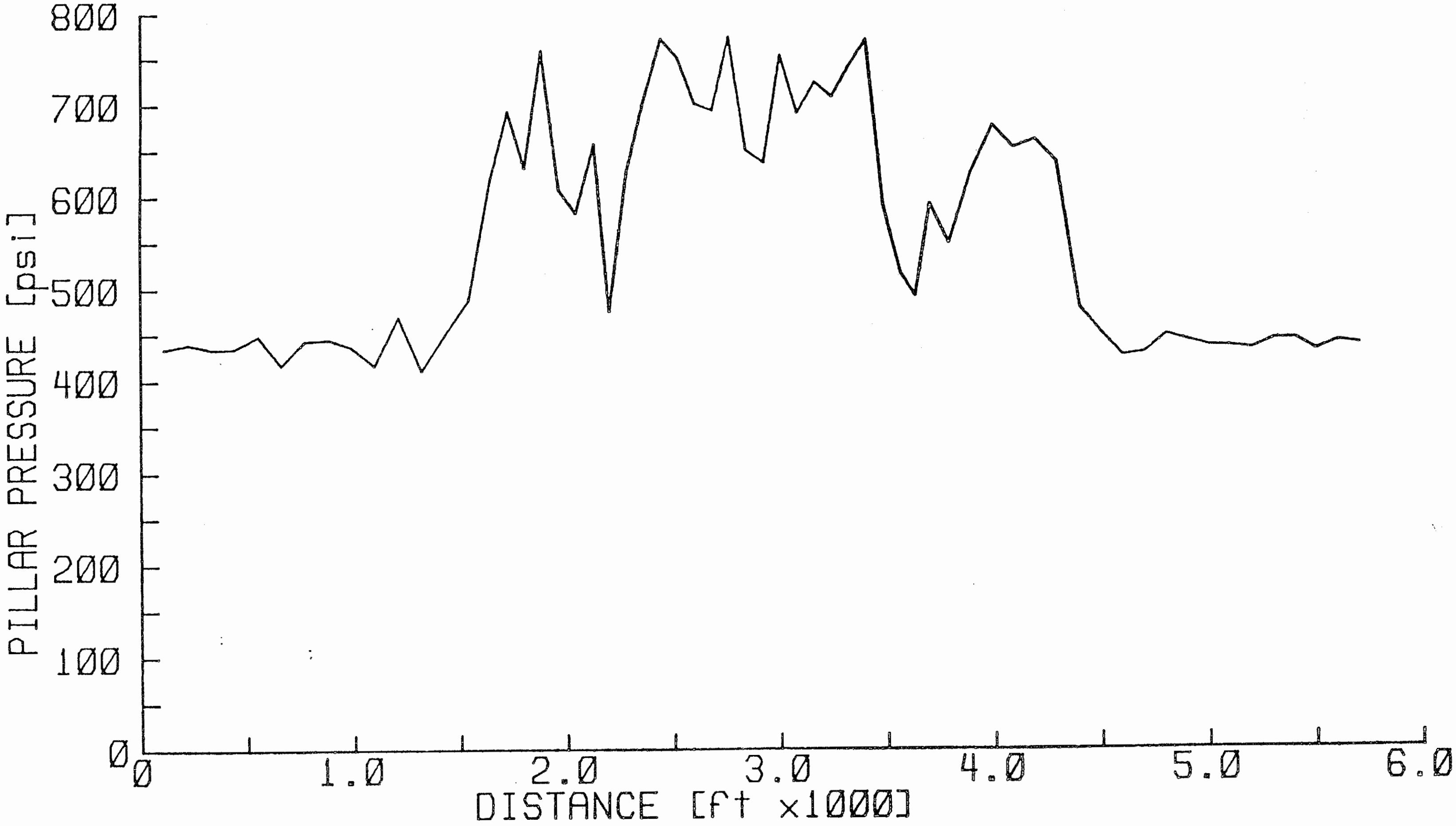


Figure II-4 Load Acting on Pillars Along Section A-A.

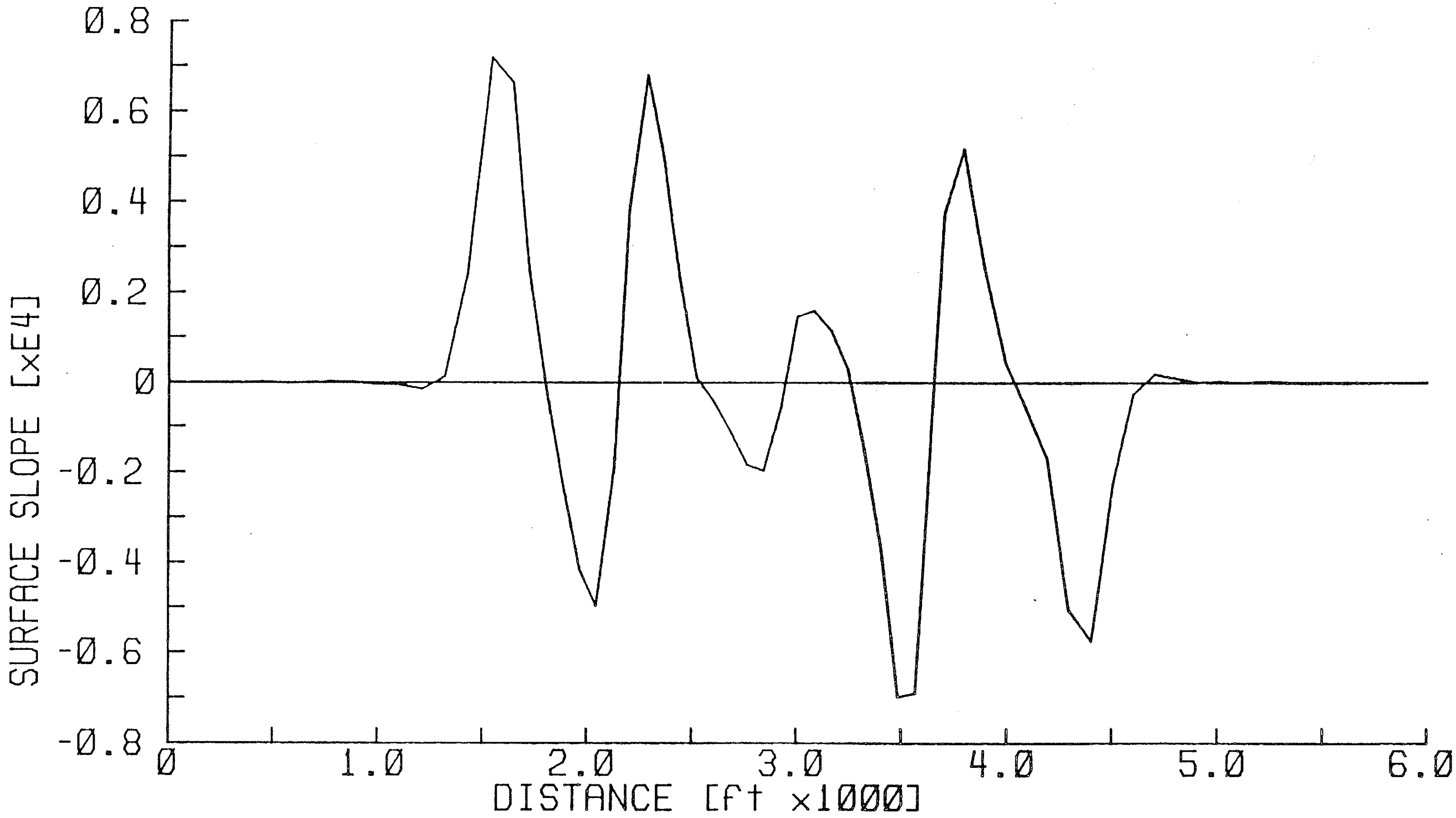


Figure II-5 Slope Along Subsidence Profile. Section A-A.

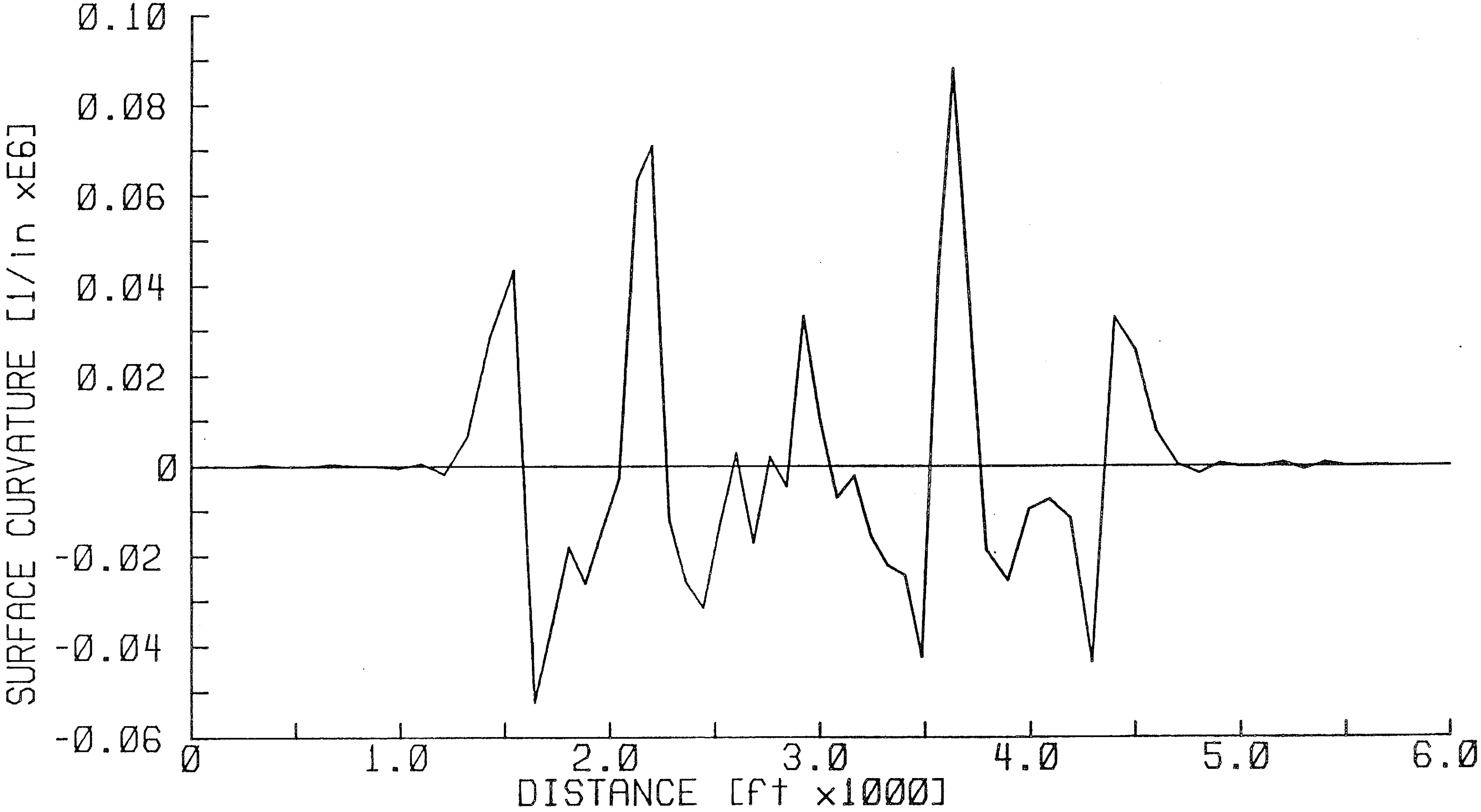


Figure II-6 Curvature Along Subsidence Profile. Section A-A.

Table II-1 Input Data For Program PANEL.2D (File INPUT.DAT)

```
94,16,28,31,63,67,79,436.7,1,0
0.35,0.35,1.E4,86.,36.21E3,40.2E3,60.35E3,1.086E8,0.
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7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
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7.273E14,2.751E8,1320.,1320.,1.E8,4.02E4,0.,1.,30.,1320.,1320.,2.E5
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7.273E14,2.751E8,216.,1104.,1.E8,4.02E4,0.,1.,30.,480.,1320.,2.E5
7.273E14,2.751E8,744.,1104.,1.E8,4.02E4,0.,1.,30.,480.,1320.,2.E5
7.273E14,2.751E8,216.,1104.,1.E8,4.02E4,0.,1.,30.,480.,1320.,2.E5
7.273E14,2.751E8,744.,1104.,1.E8,4.02E4,0.,1.,30.,480.,1320.,2.E5
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Table II-1 Cont'd

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7.273E14,2.751E8,1200.,1200.,1.E8,4.02E4,0.,1.,30.,1200.,1200.,2.E5
7.273E14,2.751E8,1200.,1200.,1.E8,4.02E4,0.,1.,30.,1200.,1200.,2.E5
1.,1.
```

Table II-2 Screen Prompts For Running PANEL.2D

```
OK, x panel.2d
[F77 Rev. 21.0.4 Copyright (c) 1988, Prime Computer, Inc.]
[Serial #IDAM-UAQ79A-C9D8 (SOUTHERN ILLINOIS UNIVERSITY)]
0000 ERRORS [<PANEL2D> F77 Rev. 21.0.4]
0000 ERRORS [<SETFIL> F77 Rev. 21.0.4]
0000 ERRORS [<OPNFIL> F77 Rev. 21.0.4]
0000 ERRORS [<MODEL> F77 Rev. 21.0.4]
0000 ERRORS [<FACT> F77 Rev. 21.0.4]
0000 ERRORS [<ROWLIN> F77 Rev. 21.0.4]
[BIND Rev. 21.0.2 Copyright (c) 1985, Prime Computer, Inc.]
BIND COMPLETE
Map of PANEL.2D

UNDEFINED SYMBOLS:
 Execution initiating

What is the name of your input  file?
input.dat


       WEAK FLOOR STRATA MODELS


         [1]  WINKLER
         [2]  ZEMOCHKIN AND SINITSYN
                (Elastic Homogeneous Half-Space)
         [3]  VLASOV
         [4]  CONFINED CLAY LAYER
         [5]
         [6]  QUIT


     WHICH NUMBER DO YOU WISH?


1

What is the name of your output file?

output.dat
 -*-
 -*-

What is the name of your settlements file?

settl

What is the name of your pressures file?

react

What is the name of your curvature file?

curve
```

Table II-2 Cont'd

```
What is the name of your slope file?

slope

              61



        WEAK FLOOR STRATA MODELS


            [1]   WINKLER
            [2]   ZEMOCHKIN AND SINITSYN
                    (Elastic Homogeneous Half-Space)
            [3]   VLASOV
            [4]   CONFINED CLAY LAYER
            [5]
            [6]   QUIT


        WHICH NUMBER DO YOU WISH?


   6
   OK, em settl
```

Table II-3 Output Data From Program PANEL.2D (File OUTPUT.DAT)

WINKLER

PILLAR No.	PRESSURE [psi]	SETTLEMENT [in]
2	0.43558E+03	-0.59009E-05
3	0.43986E+03	0.66757E-04
4	0.43420E+03	-0.14305E-03
5	0.43538E+03	0.13113E-03
6	0.44880E+03	0.73791E-04
7	0.41693E+03	-0.19825E-03
8	0.44331E+03	-0.43809E-04
9	0.44479E+03	0.16010E-03
10	0.43566E+03	0.15539E-03
11	0.41563E+03	-0.94622E-03
12	0.46848E+03	-0.13408E-02
13	0.41041E+03	-0.52030E-02
14	0.44943E+03	0.20612E-02
15	0.48753E+03	0.59188E-01
17	0.61829E+03	0.18272E+00
19	0.69188E+03	0.22223E+00
21	0.63005E+03	0.22952E+00
23	0.75805E+03	0.22019E+00
25	0.60678E+03	0.18681E+00
27	0.58067E+03	0.14053E+00
29	0.65703E+03	0.88740E-01
30	0.47458E+03	0.95622E-01
32	0.62721E+03	0.17084E+00
34	0.70304E+03	0.23059E+00
36	0.77042E+03	0.26664E+00
38	0.74980E+03	0.27363E+00
40	0.69889E+03	0.26835E+00
42	0.69194E+03	0.26563E+00
44	0.77309E+03	0.24714E+00
46	0.64879E+03	0.23035E+00

Table II-3 Cont'd

48	0.63490E+03	0.20935E+00
50	0.75214E+03	0.21884E+00
52	0.68898E+03	0.23732E+00
54	0.72223E+03	0.24928E+00
56	0.70622E+03	0.25923E+00
58	0.73957E+03	0.25469E+00
60	0.76897E+03	0.22978E+00
62	0.58942E+03	0.18239E+00
64	0.51550E+03	0.92538E-01
65	0.49023E+03	0.51320E-01
66	0.59066E+03	0.66979E-01
68	0.54805E+03	0.13595E+00
70	0.62469E+03	0.18448E+00
72	0.67500E+03	0.19644E+00
74	0.65105E+03	0.19467E+00
76	0.66030E+03	0.18222E+00
78	0.63533E+03	0.15304E+00
80	0.47709E+03	0.49811E-01
81	0.44918E+03	0.43954E-02
82	0.42461E+03	-0.43415E-02
83	0.42838E+03	-0.22712E-02
84	0.44755E+03	-0.26584E-04
85	0.44069E+03	-0.40168E-03
86	0.43464E+03	-0.47684E-06
87	0.43350E+03	-0.51618E-04
88	0.43087E+03	-0.23323E-03
89	0.44145E+03	0.40644E-03
90	0.44252E+03	-0.18954E-03
91	0.42951E+03	0.34273E-04
92	0.43913E+03	-0.14544E-04
93	0.43595E+03	0.35763E-06

Table II-4 Output File SETTL (Example)

0.110000	-5.900860E-06
0.220000	6.675720E-05
0.330000	-1.430511E-04
0.440000	1.311302E-04
0.550000	7.379055E-05
0.660000	-1.982450E-04
0.770000	-4.380941E-05
0.880000	1.600981E-04
0.990000	1.553893E-04
1.10000	-9.462237E-04
1.21000	-1.340806E-03
1.32000	-5.203009E-03
1.43000	2.061188E-03
1.54000	5.918801E-02
1.64400	0.182719
1.72400	0.222233
1.80400	0.229517
1.88400	0.220189
1.96400	0.186811
2.04400	0.140533
2.12850	8.873987E-02
2.19950	9.562230E-02
2.28400	0.170843
2.36400	0.230588
2.44400	0.266641
2.52400	0.273626
2.60400	0.268355
2.68400	0.265630
2.76400	0.247140
2.84400	0.230354
2.92400	0.209355
3.00400	0.218839
3.08400	0.237317
3.16400	0.249281
3.24400	0.259232
3.32400	0.254689
3.40400	0.229776
3.48400	0.182392
3.56650	9.253782E-02
3.63350	5.132031E-02
3.70050	6.697929E-02
3.79300	0.135954
3.89300	0.184475
3.99300	0.196439
4.09300	0.194674
4.19300	0.182218
4.29300	0.153037
4.40200	4.981059E-02
4.50200	4.395425E-03
4.60200	-4.341543E-03
4.70200	-2.271175E-03
4.80200	-2.658367E-05
4.90200	-4.016757E-04
5.00200	-4.768372E-07
5.10200	-5.161762E-05
5.20200	-2.332330E-04
5.30200	4.064441E-04
5.40200	-1.895428E-04
5.50200	3.427267E-05
5.60200	-1.454353E-05
5.70200	3.576279E-07

Table II-5 Output File REACT

0.110000	435.585
0.220000	439.865
0.330000	434.202
0.440000	435.380
0.550000	448.804
0.660000	416.935
0.770000	443.308
0.880000	444.791
0.990000	435.659
1.10000	415.633
1.21000	468.477
1.32000	410.407
1.43000	449.432
1.54000	487.531
1.64400	618.290
1.72400	691.878
1.80400	630.054
1.88400	758.050
1.96400	606.780
2.04400	580.666
2.12850	657.027
2.19950	474.585
2.28400	627.214
2.36400	703.045
2.44400	770.422
2.52400	749.805
2.60400	698.891
2.68400	691.942
2.76400	773.089
2.84400	648.791
2.92400	634.905
3.00400	752.141
3.08400	688.981
3.16400	722.231
3.24400	706.224
3.32400	739.566
3.40400	768.969
3.48400	589.416
3.56650	515.501
3.63350	490.227
3.70050	590.659
3.79300	548.045
3.89300	624.691
3.99300	675.001
4.09300	651.045
4.19300	660.297
4.29300	635.326
4.40200	477.089
4.50200	449.182
4.60200	424.605
4.70200	428.375
4.80200	447.550
4.90200	440.686
5.00200	434.642
5.10200	433.499
5.20200	430.866
5.30200	441.446
5.40200	442.517
5.50200	429.511
5.60200	439.129
5.70200	435.950

Table II-6 Output File SLOPE

0.110000	4.111365E-04
0.220000	-5.195087E-04
0.330000	2.438372E-04
0.440000	8.213700E-04
0.550000	-1.247634E-03
0.660000	-4.454545E-04
0.770000	1.357360E-03
0.880000	7.545404E-04
0.990000	-4.190614E-03
1.10000	-5.667410E-03
1.21000	-1.612419E-02
1.32000	1.288634E-02
1.43000	0.243905
1.54000	0.719113
1.64400	0.663008
1.72400	0.243743
1.80400	-1.064688E-02
1.88400	-0.222426
1.96400	-0.414875
2.04400	-0.496031
2.12850	-0.189320
2.19950	0.382608
2.28400	0.680450
2.36400	0.498944
2.44400	0.224153
2.52400	8.927347E-03
2.60400	-4.164503E-02
2.68400	-0.110492
2.76400	-0.183727
2.84400	-0.196798
2.92400	-5.997284E-02
3.00400	0.145637
3.08400	0.158549
3.16400	0.114140
3.24400	2.816474E-02
3.32400	-0.153419
3.40400	-0.376542
3.48400	-0.697414
3.56650	-0.689664
3.63350	-0.158946
3.70050	0.373974
3.79300	0.517096
3.89300	0.252022
3.99300	4.249413E-02
4.09300	-5.925472E-02
4.19300	-0.173486
4.29300	-0.504427
4.40200	-0.574983
4.50200	-0.225634
4.60200	-2.777751E-02
4.70200	1.797899E-02
4.80200	7.789581E-03
4.90200	1.087784E-04
5.00200	1.458575E-03
5.10200	-9.698174E-04
5.20200	1.908590E-03
5.30200	1.820425E-04
5.40200	-1.550714E-03
5.50200	7.291633E-04
5.60200	-1.413127E-04
5.70200	7.823108E-05

Table II-7 Output File CURVE

0.110000	2.110655E-05
0.220000	-1.621134E-04
0.330000	2.777718E-04
0.440000	-1.902669E-04
0.550000	-1.232185E-04
0.660000	2.447607E-04
0.770000	2.839293E-05
0.880000	-1.197293E-04
0.990000	-6.295366E-04
1.10000	4.057795E-04
1.21000	-1.990140E-03
1.32000	6.385673E-03
1.43000	2.861720E-02
1.54000	4.338393E-02
1.64400	-5.237510E-02
1.72400	-3.497181E-02
1.80400	-1.802601E-02
1.88400	-2.609462E-02
1.96400	-1.399901E-02
2.04400	-2.908486E-03
2.12850	6.340370E-02
2.19950	7.085161E-02
2.28400	-1.210555E-02
2.36400	-2.570838E-02
2.44400	-3.153976E-02
2.52400	-1.329890E-02
2.60400	2.762923E-03
2.68400	-1.710607E-02
2.76400	1.848740E-03
2.84400	-4.571823E-03
2.92400	3.307707E-02
3.00400	9.758253E-03
3.08400	-7.068156E-03
3.16400	-2.183758E-03
3.24400	-1.572784E-02
3.32400	-2.210223E-02
3.40400	-2.438170E-02
3.48400	-4.246654E-02
3.56650	4.403199E-02
3.63350	8.798739E-02
3.70050	4.457975E-02
3.79300	-1.879200E-02
3.89300	-2.538715E-02
3.99300	-9.534094E-03
4.09300	-7.424048E-03
4.19300	-1.161446E-02
4.29300	-4.354239E-02
4.40200	3.275388E-02
4.50200	2.547096E-02
4.60200	7.505092E-03
4.70200	1.209891E-04
4.80200	-1.819225E-03
4.90200	5.390907E-04
5.00200	-3.141248E-04
5.10200	-9.060734E-05
5.20200	5.703417E-04
5.30200	-8.580999E-04
5.40200	5.693069E-04
5.50200	-1.893275E-04
5.60200	4.424816E-05
5.70200	-7.657542E-06