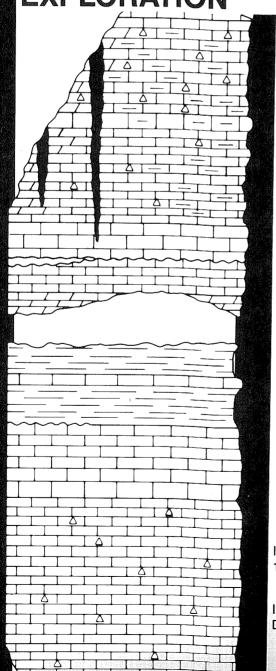


A GRAVITY SURVEY OF MARINE FIELD: CASE STUDY FOR SILURIAN REEF EXPLORATION



Paul C. Heigold and Stephen T. Whitaker

Illinois Petroleum 133 1989

ILLINOIS STATE GEOLOGICAL SURVEY Department of Energy and Natural Resources

A GRAVITY SURVEY OF MARINE FIELD: CASE STUDY FOR SILURIAN REEF EXPLORATION

Paul C. Heigold and Stephen T. Whitaker

Illinois Petroleum 133 1989

ILLINOIS STATE GEOLOGICAL SURVEY Morris W. Leighton, Chief Natural Resources Building 615 East Peabody Drive Champaign, IL 61820

CONTENTS

AE	BSTRACT	1
IN'	TRODUCTION	1
GE	EOLOGIC DISCUSSION OF MARINE FIELD	3
IN'	TERPRETATION OF GRAVITY DATA AT MARINE FIELD	7
Во	ouguer gravity anomaly	9
Re	esidual Bouguer gravity anomaly	11
Se	econd vertical derivative	12
SF	PACING OF STATIONS IN A GRAVITY SURVEY	13
SL	JMMARY	15
RE	EFERENCES	15
AF	PPENDIX A. TREND SURFACE ANALYSIS	17
AF	PPENDIX B. SECOND VERTICAL DERIVATIVE	18
FIC	GURES	
1	. Regional map showing Silurian reefs in Illinois	2
2	. Silurian structure map at Marine Field	4
3	. Generalized stratigraphic column of southern Illinois	5
4	. Schematic cross sections illustrating effects of differential compaction around a buried reef	6
5	. Schematic cross section showing average density values of strata in and around the Silurian reef	7
6	Bouguer gravity anomaly map and structure map on the Ste. Genevieve Formation at Nashville Reef	8
7	. Bouguer gravity anomaly at Marine Field	9
8	. First order least squares trend surface at Marine Field	12
9	. Residual Bouguer gravity anomaly surface at Marine Field	13
10	. Second vertical derivative residual Bouguer gravity anomaly map at	4 4

A GRAVITY SURVEY OF MARINE FIELD: A CASE STUDY FOR SILURIAN REEF EXPLORATION

Paul C. Heigold and Stephen T. Whitaker

ABSTRACT

A gravity survey conducted over and around Marine Field in southwestern Illinois has been used as an example to show how measurement of the local gravity field can aid in the search for Silurian reefs and possible associated hydrocarbon reservoirs in the Illinois Basin. Acquisition parameters for gravity surveys over Silurian reefs should be calculated beforehand from simple models of the reef based on reasonable estimates of density contrasts, depths, and size. These calculations can ensure that the spacing of the stations will be close enough to maximize any mappable anomaly and prevent erroneous interpretations of anomaly size and shape. A gridded pattern of properly spaced stations will further enhance the ability of the geologist or geophysicist to generate accurate maps and facilitate subsequent manipulation of the data.

Residual and derivative mapping techniques generally enhance gravity anomalies and enable more accurate portrayals of the structural relief on buried reefs. The second vertical derivative map of the residual Bouguer gravity anomaly surface at Marine compares well with the configuration of the reef as mapped from oil well data. This study indicates that similar mapping techniques could be effective on other reefs throughout the Illinois Basin. Gravity surveys are not a panacea for reef exploration, however, because they cannot detect reefs in which there are extremely small, lateral, density contrasts with adjacent strata nor can they always differentiate between reef anomalies and anomalies due to other geologic factors.

Although gravity mapping methods are potentially powerful exploration tools in themselves, the authors believe that the proper role of these methods is as part of a larger exploration scheme. Gravity surveys can be used effectively as an initial exploration method in reef-prone areas to define smaller, prospect-size areas in which geologists or geophysicists can target locally intensive exploration techniques.

INTRODUCTION

Oil exploration for Silurian reefs in Illinois has been generally limited to a northeast-southwest trend in the south central part of the state (fig. 1). This trend has netted approximately 33 million barrels of oil from reef rock itself and another 61 million barrels of oil from younger strata draped over the reefs. Despite past successes, no Silurian pinnacle reefs have been discovered since 1974 in Illinois. This fact suggests either that there are not more Silurian reefs to be found in the state or that exploration strategies are not adequate.

Recent articles by Coburn (1986) and Whitaker (1988a,b) have stressed the importance of exploring for Silurian pinnacle reefs across a more extensive part of Illinois than that indicated by the present trend. Efforts for locating buried pinnacle

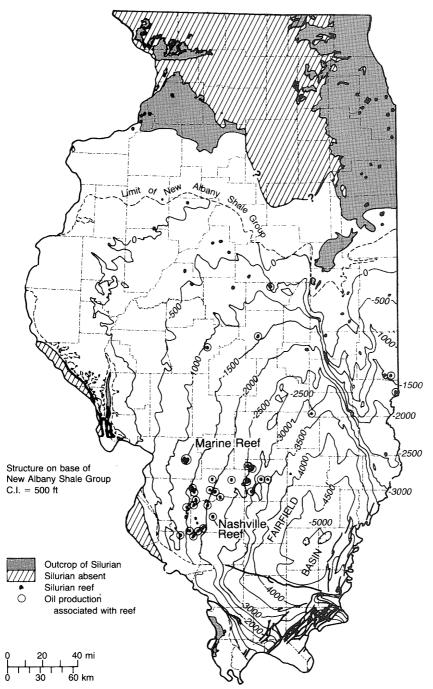


Figure 1 Regional map showing Silurian reefs in Illinois.

reefs in the future will require utilization of economical and accurate exploration programs. Detailed gravity surveying could prove to be a useful part of such programs.

This paper presents data gathered from a gravity survey performed over and around the reef underlying Marine Field in Madison County, Illinois. These data are analyzed in order to examine further the validity of the gravity method as a tool in the search for pinnacle reefs throughout the basin. Lowenstam (1949, p. 35) noted that "Gravity anomalies have proved successful in locating Permian reefs in west Texas, but it is not known whether there is sufficient difference in the density of reef and interreef

Niagaran sediments to produce recognizable anomalies." Subsequently, Ferris (1972) and Dana (1980) documented gravity anomalies over reefs in the Illinois Basin. This paper provides further documentation and elucidates useful methodology in adapting gravity exploration techniques to the search for buried reefs.

GEOLOGIC DISCUSSION OF MARINE FIELD

Marine Field, located in sections 3, 4, 5, 8, 9, 10, 11, 15, 16, and 17, T4N-R6W, Madison County, Illinois (fig. 1), was discovered primarily on the basis of seismic reflection data. The discovery well, the Eason-Rockhill-Obering Mayer No.1 (SE NW SW sec. 15, T4N-R6W), was completed in July, 1943, in Silurian carbonates at a depth of 1736 feet. The discovery of this field, the first to produce commercial quantities of oil from Silurian reef rock in Illinois, initiated an earnest search for similar reservoirs. Total production from Marine Field is approximately 12.5 million barrels of oil, almost entirely from Silurian reef rock.

Lowenstam and duBois (1946) and Lowenstam (1948) were the first to describe the geology of Marine Field. It is not the purpose of this paper to duplicate the detail of their work; however, geologic features common to Silurian pinnacle reefs that affect the earth's gravity field warrant a brief discussion.

Silurian reefs in the Illinois Basin are commonly less than a mile in diameter; however, the reef responsible for Marine Field encompasses portions of 9 square miles, making it the largest in areal extent of any known Silurian reef in the basin (fig. 2). As is commonly the case with reefs in the region, reef growth at Marine Field apparently began in early Niagaran time (fig. 3) and continued through Cayugan time, with minor interruptions due to fluctuating sea levels. A major regression that began during early Devonian time initiated erosion not only of undetermined amounts of any Lower Devonian strata that had been deposited in the deeper parts of the proto-Illinois Basin (Whitaker, 1988b), but also of Silurian strata from a large part of Illinois, including the Marine Reef area. This erosion left approximately 570 feet of the reef intact at Marine covered by a thin mantle (2 to 40 ft) of carbonate rubble. Rocks of Middle Devonian age unconformably overlie the reef.

Reef facies differ appreciably from regional nonreef Silurian strata and commonly contribute to any density contrast between reef and interreef rocks. Reef rocks are typically 90 percent or more carbonates with corresponding low insoluble residues, whereas all regional Niagaran and Cayugan rocks contain less than 90 percent carbonates and commonly contain less than 80 percent (Lowenstam, 1948). Chert is conspicuously absent in reef facies, although common in the interreef carbonates.

The clean carbonates that make up the reef core are typically massive and contain abundant bioclastic debris. At Marine Field, the reef core consists of a dark bluish gray, vuggy, dense to finely sucrosic dolomite. The effects of dolomitization, probably due to subaerial exposure of the reef during the Devonian and the subsequent effects of groundwater on the reef carbonates, appear to decrease with increasing distance from the reef core. Visible porosity is generally developed around fossil cavities that were enlarged by solution and along the numerous intersecting fractures commonly found in reef cores.

The horseshoe outline of Marine Reef and its similarity to outlines of modern reefs in Australia led Lowenstam (1950) to conclude that the reef was formed in the unobstructed path of the prevailing winds as it grew upward to wave base. The two northward trending windswept horns of Marine Reef consist mostly of bioclastic debris that was washed off the growing reef center. Small satellite reefs developed locally on these horns and further extended growth of the reef complex.

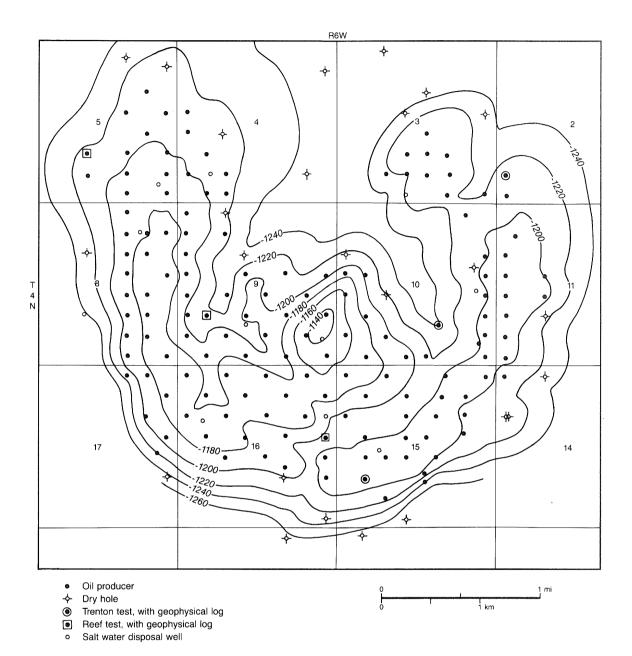


Figure 2 Silurian structure map at Marine Field.

Lowenstam (1948) noted that the reef complex at Marine Field is mantled by coarsely crystalline limestone deposits consisting of skeletal coquinas. This mantle varies in thickness from 2 to 40 feet and contains abundant crinoidal fragments, heavy shelled brachiopods, corals, stromatoporoids, and bryozoans. Visible porosity consists of fossil cavities, interskeletal spaces of colonial corals, interstices of the coquina, and discontinuous fissures. The flanks of the reef are predominantly dolomitic limestones and limestones that locally contain patches of dolomites. The purer flank limestones are densely packed aggregates of fossil skeletons that are similar to texture and composition to the reef-capping detritus.

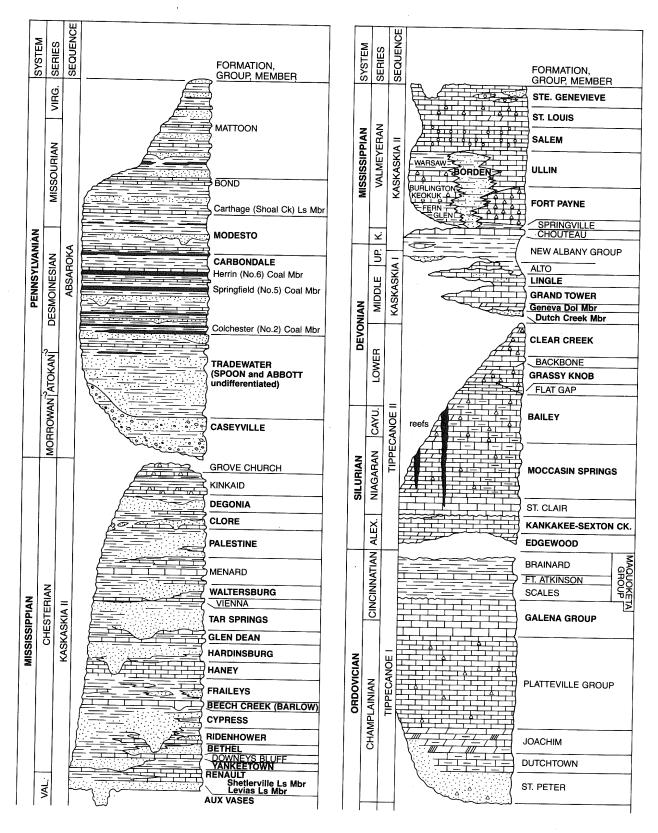


Figure 3 Generalized stratigraphic column of southern Illinois. Only Paleozoic strata younger than Champlainian are shown. Thicknesses are not to scale. Formations that contain hydrocarbon reservoirs are shown in bold type. The series names Alexandrian, Cayugan, Upper (Devonian), Kinderhookian, Valmeyeran, and Virgilian are abbreviated Alex., Cayu., Up., K., Val., and Virg., respectively.

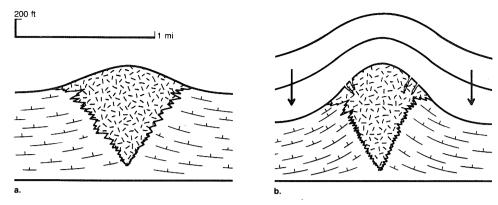


Figure 4 Schematic cross sections illustrating effects of differential compaction around a buried reef: (a) during deposition, relief from reef top to sea floor was probably a few tens of feet (Droste and Shaver, 1987; Whitaker, 1988a,b; (b) interreef lime muds have been compressed because of the weight of overburden, whereas the rigid framework of reef core resisted compaction. This differential compaction has resulted in exaggeration of reef structure and has caused draping of younger strata over the reef. Note fractures in reef edges due to compaction on interbedded reef-interreef beds.

The interreef rock was deposited in relatively quiet water and contrasts vividly with reef core and reef flank facies. With increasing distance from the reef, less reef-derived detritus was deposited and more clay and silt were incorporated into the fine limestone. The gradually expanding reef complex influenced sedimentation within a 2-mile radius (Lowenstam, 1950). The intertonguing of reef detritus and interreef rock created a complex pattern of lithologies around the reef.

The structural relief associated with the reef at Marine, like that associated with many other buried reefs, is due primarily to a phenomenon known as differential compaction (Nevin and Sherill, 1929). Droste and Shaver (1987) and Whitaker (1988a,b) have stated that the original relief from the top of Silurian reefs to the adjacent sea floor was perhaps only a few tens of feet. The weight of the overburden on the Silurian strata caused compaction of the soft interreef lime muds around the reef, whereas the rigid framework of the reef core itself resisted compaction. This differential compaction has resulted in the draping of younger strata over the reef to form a compaction anticline (dome) (fig. 4). Preliminary findings by the authors suggest that structural draping over pinnacle reefs is a prerequisite for the existence of an observable gravity anomaly.

One of the difficulties with analyzing gravity responses to buried reefs in the Illinois Basin is the sparsity of density data with which to calibrate the gravity. Dana (1980) analyzed the gravity anomaly over Wilfred Pool in Sullivan County, Indiana, by measuring the density of selected rock samples in the laboratory. He concluded that the reef core itself did not contribute substantially to the gravity anomaly there because there was not sufficient density contrast between the reef core and adjacent strata. Instead, it appeared that the gravity anomaly at Wilfred Pool is caused by density contrasts between formations draped over the buried reef.

As a supplement to Dana's work, the authors are examining Nashville Reef (secs. 19 and 20, T2S-R3W, Washington County, Illinois, 32 miles southeast of Marine Field) because there are sufficient density logs there for analysis. Interpretations of the information available on Nashville Reef indicate that there is little density contrast between the reef core and adjacent Silurian or Devonian carbonates (fig. 5). Likewise, there is only minimal density contrast between Devonian or Mississippian carbonates and Silurian strata. Although there is a contrast between the density of the Devonian New Albany Shale and overlying or underlying carbonates, the interval is not sufficiently thick to provide a gravity anomaly. The most significant density contrast at Nashville Reef, and presumably at other buried Silurian pinnacle

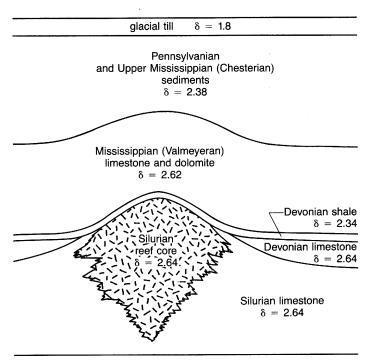


Figure 5 Schematic cross section showing average density values of strata in and around Nashville Reef, Washington County, Illinois.

reefs in the basin as well, occurs between Valmeyeran (middle Mississippian) strata, which are mostly carbonates, and Chesterian (upper Mississippian) strata, which are mostly siliciclastics (fig. 5). It is the pronounced doming of the relatively dense Valmeyeran carbonates and their structural relationship to the less dense Chesterian strata that apparently produces most of the observed gravity anomaly (fig. 6).

Further studies of gravity anomalies at other buried Silurian reefs in the Illinois Basin are planned to determine the applicability of the examples at Wilfred Pool and Nashville Reef to other reefs. Because of similarities in the stratigraphy above and adjacent to Marine Reef and Nashville Reef, it is probably safe to assume that the gravity anomalies at these two reefs are caused by the same effects of structural drape.

INTERPRETATION OF GRAVITY DATA AT MARINE FIELD

Gravity surveying involves observing the acceleration of gravity, g, at a number of discrete points, usually located at or near the earth's surface, in an area of interest. After correcting these observations for instrumental drift, "free-air" and "Bouguer" corrections are made which account for the difference in elevation and for the mass between the observation points and a reference level, respectively. These corrections provide the *Bouguer gravity values*. In order to account for variations in latitude, theoretical gravity values, like those given by the 1930 International Formula, are subtracted from the Bouguer gravity values and thus provide the *Bouguer gravity anomaly values* (Nettleton, 1940). Variations in Bouguer gravity anomaly values are the result of lateral density contrasts within the earth. It is the task of the interpreter to isolate and analyze the variations in Bouguer gravity anomaly values due to lateral variations in density associated with a geologic target of interest, such as a reef.

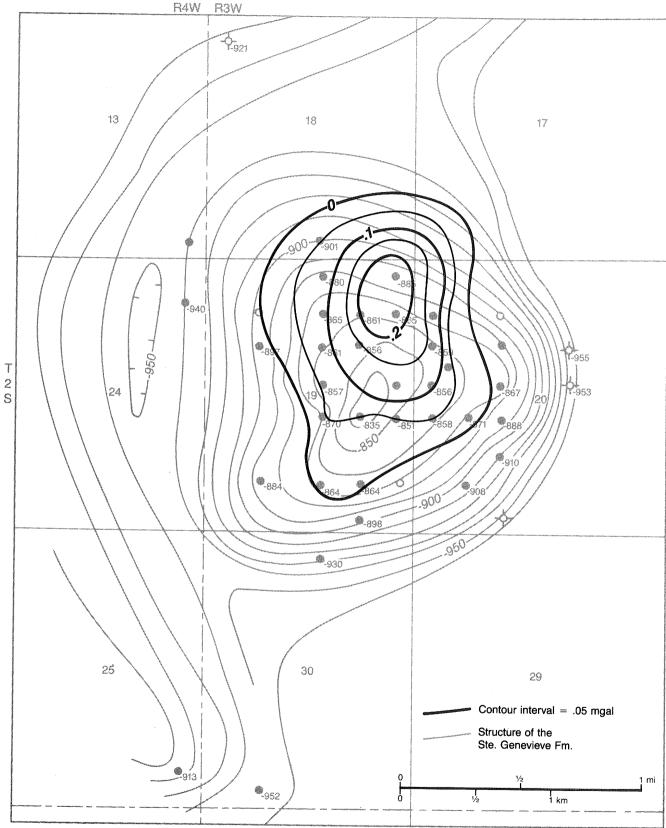


Figure 6 Bouguer gravity anomaly map and structure map on the Ste. Genevieve Formation at Nashville Reef, Washington County, Illinois. Gravity interpretation courtesy of Leonard Rosenfeld, consulting geophysicist.

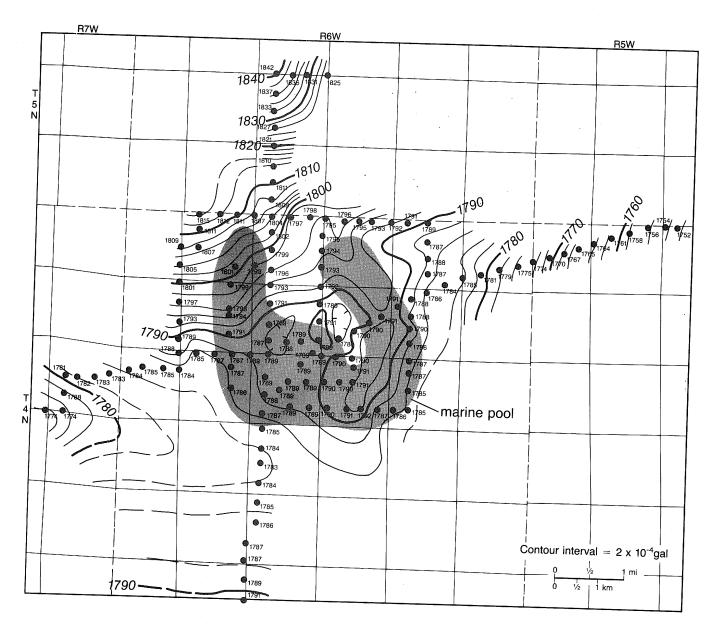


Figure 7 Bouguer gravity anomaly at Marine Field.

Bouguer gravity anomaly

Figure 7 is a Bouguer gravity anomaly map based on 138 gravity stations over and around Marine Reef. Gravity stations are located along available roads and station spacings are generally one quarte: of a mile. Although most of the gravity stations are concentrated over the reef, several lines extend away from the reef in order to determine the regional trend in the Bouguer gravity anomaly data. The individual Bouguer gravity anomaly values are accurate to the nearest 0.01 milligals (1 gal=1 cm/sec²), and the contour interval on figure 7 is 0.20 milligals.

In general, the Bouguer gravity anomaly surface (fig. 7) correlates with the structural configuration of the reef at Marine Field (fig. 2). A positive closure on the Bouguer gravity anomaly map is associated with the southeast part of the reef. This positive closure corresponds to the extension of a positive structural nose over the northeastern part of the reef. A much weaker positive nose on the Bouguer gravity

anomaly map corresponds to the western leg of the horseshoe-shaped reef. The smaller positive closure at stations 115 and 1108 corresponds closely with the structural high on the top of the Silurian in the southeastern part of section 9, T4N-R6W.

The degree to which the Bouguer gravity anomaly data correlate with the structural data on the top of the Silurian is adequate. Although this observation is not surprising, it is gratifying because the gravity data contain not only reef-related components but also components from other possible sources. These possible sources include: (1) structures, unconformities, and lithologic changes in the sedimentary column; (2) relief on the basement surface; and (3) lateral density changes in the crystalline portion of the earth's crust and upper mantle. Any combination of these sources could provide density contrasts between adjacent strata and thus result in a gravity anomaly. Normally these sources cause gravity anomalies on a much larger, commonly regional, scale than those due only to Silurian reefs; however, these large-scale features may still adversely affect the correlation between the reef structure and a gravity anomaly over the reef.

A previous study by Dana (1980) has indicated that any components of the Bouguer gravity anomaly data that are reef related are primarily the result of lateral density contrasts between rigid reef core and the surrounding rocks, plus the integrated effects of the lateral density contrasts from a compaction-induced anticline above the reef. These contrasts control the sign of the reef-related component of the Bouguer gravity anomaly data. Where compaction anticlines or domes are well-developed over a reef and denser Valmeyeran carbonates are adjacent to younger, less dense Chesterian siliciclastics (fig. 5), the reef-related components of the Bouguer gravity anomaly values over the reef would be positive. Previous studies by Ferris (1972) and Dana (1980) have documented positive anomalies over several buried pinnacle reefs in the Illinois Basin.

The algebraic sum of all reef-related components of the Bouguer gravity anomaly values could also be negative. This situation could exist if there were little or no structural draping over a reef (because of a lack of differential compaction) and if the reef rock itself were less dense (more porous) than the surrounding rock. Although this type of gravity anomaly over a reef has not been identified in the basin yet, it is possible that negative Bouguer gravity anomalies would exist in parts of the basin where Devonian erosion and subsequent dolomitization has prevented differential compaction from taking place, thus precluding the formation of domes or anticlines over buried reefs (Whitaker, 1988a,b).

In order to isolate those portions of the Bouguer gravity anomaly data that are reef related, the authors used the following rationale. Investigation of the regional geology of the study area indicated that with the exception of components due to the reef or to the drift-bedrock unconformity, there were no structural or lithological anomalies that could be sources for gravity changes. Consequently, all other parts of the Bouguer gravity anomaly that are due to possible geologic sources mentioned earlier could be lumped into a regional component. This procedure is reasonable in light of the fact that even Marine Reef, the largest known reef in Illinois, has a diameter of only 3 miles and therefore has a limited influence on the regional trend.

The component of the Bouguer gravity anomaly data resulting from lateral density contrasts at the drift-bedrock interface usually appears as short wave-length anomalies. These anomalies are best developed over drift-filled bedrock valleys (McGinnis et al., 1963). As with the regional component of the Bouguer gravity anomaly data, the component related to the drift-bedrock interface must be evaluated and, if necessary, removed from the data in order to isolate the reef-related component. According to Horberg (1957), the bedrock surface in T4N-R6W,

Madison County, Illinois, is relatively flat and contains no drift-filled bedrock valleys. Therefore, those components of the Bouguer gravity anomaly data resulting from any lateral density contrasts between the bedrock surface and the overlying glacial till would most likely be negligible.

The Bouguer gravity anomaly surface g(x,y) was therefore separated into two parts: a regional Bouguer gravity anomaly surface, $g^*(x,y)$, and a residual Bouguer gravity anomaly surface, $\Delta g(x,y)$, which corresponds to the reef-related components of the Bouguer gravity anomaly data:

$$g(x,y) = g^*(x,y) + \Delta g(x,y)$$

Equation (1)

Using trend surface analysis, the theoretical regional Bouguer gravity anomaly surface was calculated as shown in appendix A.

Because Silurian reefs in the Illinois Basin are commonly less than a mile in diameter and are not known to exceed 3 miles in diameter, the areal extent of a gravity survey over a reef can be relatively limited. In general, a gravity survey used for evaluating the regional trend should extend at least a mile or two beyond the limits of a reef. Once acquired, a low order polynomial trend surface should adequately represent the regional component of the Bouger gravity anomaly data.

Residual Bouguer gravity anomaly

Figure 8 illustrates the first order (planar) least-squares trend surface representing the regional component of the Bouguer gravity anomaly surface in the vicinity of Marine Reef. This plane dips south 29.52° east at 0.85 milligals per mile. Although this trend surface is as accurate as can be expected when this data set is used, a more accurate representation of the regional component near Marine Reef would have been possible if more gravity data had been gathered away from the reef itself. This first-order surface is thought to be the best representation of the regional component of the Bouguer gravity anomaly surface because in this particular survey a higher order trend surface would be too strongly influenced by the preponderance of data over the reef itself.

The residual Bouguer gravity anomaly surface, g(x,y), over Marine Reef can now be calculated (fig. 9). A sizable low is associated with the reef's lagoon (fig. 2), which is open to the north. Positive noses are in evidence over the eastern and western portions of the horseshoe-shaped reef. The residual Bouguer gravity anomaly values drop off sharply on the southwestern side of the reef.

Comparison of the residual Bouguer gravity anomaly surface, $\Delta g(x,y)$ (fig. 9), with the Bouguer gravity anomaly surface, g(x,y) (fig. 7), illustrates that the positive closures present on the Bouguer gravity anomaly surface over the eastern leg of the reef and in the southeastern part of section 9, T4N-R6W have been reduced to positive noses on the residual Bouguer anomaly surface. The weak positive nose shown in figure 7 over the western leg of the reef, however, has been amplified to a stronger positive nose in figure 9. In general, the residual Bouguer gravity anomaly surface over Marine Reef does not present a noticeably superior enhancement of the Bouguer gravity anomaly surface. However, the generation of this map is a necessary step for further study of the gravity anomaly.

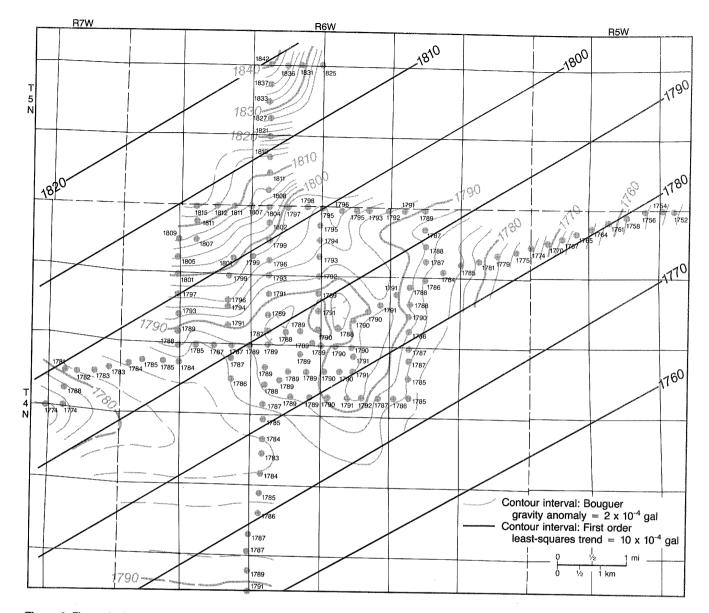


Figure 8 First order least-squares trend surface overlaid on Bouguer gravity surface.

Second vertical derivative

Reef-related anomalies present on the residual Bouguer gravity anomaly surface, $\Delta g(x,y)$, can be further enhanced by using second vertical derivative, $(\delta^2 \Delta g/\delta z^2)$ techniques described in appendix B. These derivatives are essentially a measure of the curvature of the residual Bouguer gravity anomaly surface.

Figure 10 illustrates the map of the second vertical derivative, $\delta^2 \Delta g/\delta z^2$, of the residual Bouguer gravity anomaly surface over Marine Reef. The geometry of several sharply delineated areas having positive second vertical derivative values in this figure correspond with the eastern and western legs of the horseshoe-shaped structure and with a structural high in the southeastern part of section 9, T4N-R6W. Moreover, areas with negative second vertical derivative values correspond to the lagoonal area on the structure map (fig. 2). Although a thin band of large positive second vertical derivative values exists along the southern part of the reef structure, this high area is undoubtably affected by the finite difference calculations resulting

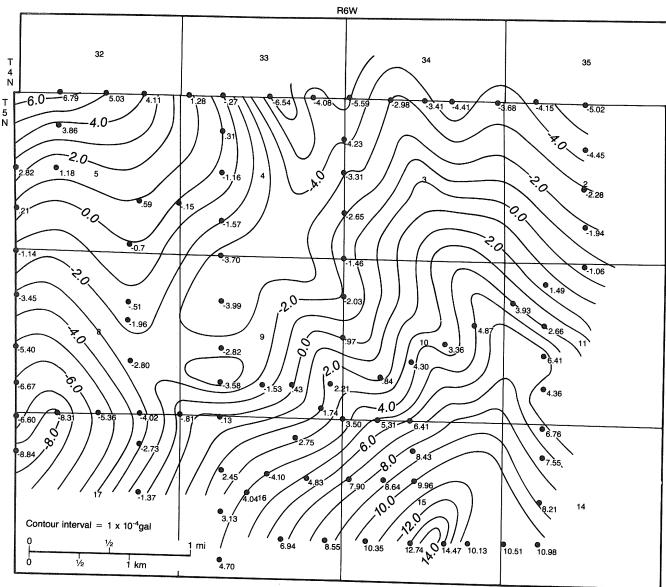


Figure 9 Residual Bouguer gravity anomaly surface at Marine Field.

from a lack of gravity data immediately to the south. A more accurate representation of the reef structure would have been possible had the gravity survey been expanded beyond the limits of the reef.

The lateral deviations of areas with positive and negative second vertical derivative values in figure 10 from regions that are structurally high and low in figure 2 may be the result of lateral density contrasts elsewhere in the sedimentary column, probably shallower than the reef. Nevertheless, the orientation and overall configuration of the reef are approximated well by the second vertical derivative values.

SPACING OF STATIONS IN A GRAVITY SURVEY

The nodal spacing used in gridded residual Bouguer gravity anomaly data must be small enough to ensure that no information on the residual Bouguer gravity anomaly surface, $\Delta g(x,y)$, is lost. The sampling theorem states: "No information is lost by regular sampling provided that the sampling frequency is greater than twice the highest frequency component in the waveform being sampled" (Telford et al., 1976).

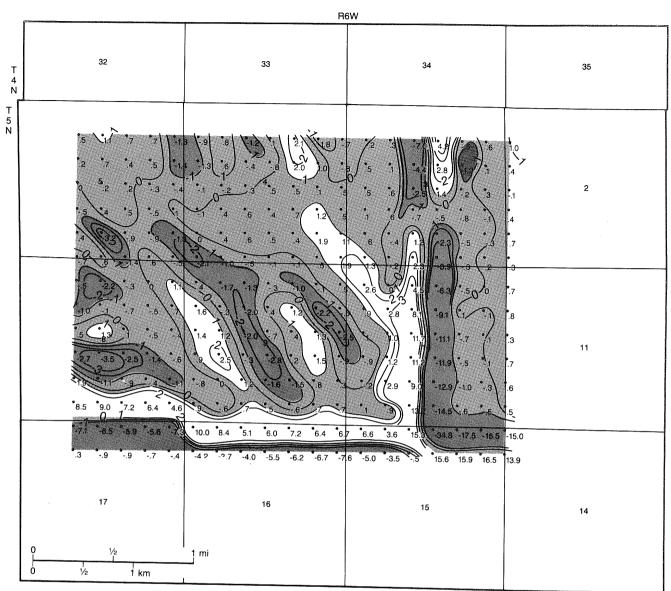


Figure 10 Second vertical derivative residual Bouguer gravity anomaly map at Marine Field (white areas = > 1 unit, light gray areas = -1 to 1 unit, and dark gray areas = < -1 unit).

This theorem states not only that a minimum sampling frequency can be determined for any given anomaly, but also that this minimum sampling frequency allows complete recovery of the anomaly being examined and that therefore, nothing can be gained by using a finer sampling density.

Great savings in time and money can be made during the data acquisition, reduction, and interpretation phases of a program to explore for reefs if one has some prior knowledge of the wavelength of the reef-related anomaly that might be present on the Bouguer gravity anomaly surface. Such knowledge will permit gravity data to be gathered on a grid having optimal nodal spacing for subsequent manipulation of these data. One of the many advantages of having gridded sets of data is the fact that machine contouring algorithms produce the most reliable maps from such input.

Some idea of the wavelength of the possible reef-related gravity anomaly, and therefore the proper grid size for collecting data, can be derived before the data-collection phase of a gravity study by calculating gravity anomalies caused by

theoretical bodies. These theoretical bodies can be of rather simple geometric shapes; cylinders tend to approximate pinnacle reefs well. The pertinent parameters, such as depth, shape, size, and density contrast, can be varied with little effort over geologically reasonable ranges, and corresponding anomalies can be calculated and examined (Nettleton, 1940).

SUMMARY

Acquisition, processing, and interpretation of gravity data at Marine Field indicate that the gravity survey in this study accurately and economically mapped the buried pinnacle reef there. The second derivative map of the residual Bouguer anomaly gravity surface was the most easily visualized indicator of the buried pinnacle reef at Marine Field. Although maps generated from the gravity survey in this study compared extremely well to the actual structure on the reef at Marine Field, an even better correlation would have been possible with better nodal spacing and station layout that took into account the expected size of the reef.

Preliminary results from a study being conducted by the authors suggest that positive gravity anomalies associated with buried Silurian pinnacle reefs in the Illinois Basin will be observed only in those areas that contain reef-associated domes caused by differential compaction. Therefore, exploration methods other than gravity will have to be utilized in those parts of the basin in which pre-Middle Devonian erosion and dolomitization has precluded reef-induced structures (Whitaker, 1988 a,b).

Analyses similar to those performed in this study can be applied to other gravity surveys and may enhance exploration for pinnacle reefs throughout pertinent portions of the Illinois Basin. Logically, gravity surveying should follow isolation of prospective areas on the basis of known geology or regional geophysical surveys and precede more expensive exploration techniques such as detailed seismic reflection profiling and drilling.

REFERENCES

- Bristol, H.M., 1974, Silurian pinnacle reefs and related oil production in southern Illinois: Illinois State Geological Survey Illinois Petroleum 102, 98 p.
- Coburn G.W., 1986, Silurian of the Illinois basin: a carbonate ramp: Oil and Gas Journal, v. 84, no. 40, p. 96-100.
- Coons, R.L., J.W. Mack, and W. Strange, 1964, Least-square polynomial fitting of gravity data and case histories of computers in the mineral industries: School of Earth Science, Stanford University, p. 498-519.
- Dana, S.W., 1980, Analysis of gravity anomaly over coral-reef oil field: Wilfred Pool, Sullivan County, Indiana: American Association of Petroleum Geologists Bulletin, v. 64, no. 3, p. 400-413.
- DeBoor, C., 1962, Bicubic spline interpolation: Journal of Mathematical Physics v. 41, no. 3, p. 212-218.
- Droste, J.B., and R.H. Shaver, 1987, Upper Silurian and Lower Devonian stratigraphy of the central Illinois Basin: Indiana Geological Survey Special Report 39, 29 p.
- Ferris, C., 1972, Use of gravity meter in search for stratigraphic traps, in Stratigraphic oil and gas fields-classification, exploration methods, and case histories: American Association of Petroleum Geologists Memoir 16, p. 252-267.
- Horberg, L., 1957, Bedrock surface of Illinois: Illinois State Geological Survey map, second edition (scale, 1:500,000).

- Lowenstam, H.A., 1948, Marine Pool, Madison County, Illinois, Silurian reef producer, *in* J.V. Howell [ed.], Structure of typical American oil fields, v. 3: American Association of Petroleum Geologists, p. 153-188: Illinois State Geological Survey Report of Investigations 131, 36 p.
- Lowenstam, H.A., 1949, Niagaran reefs in Illinois and their relation to oil accumulation: Illinois State Geological Survey Report of Investigations 145, 40 p.
- Lowenstam, H.A., 1950, Niagaran reefs of the Great Lakes area: Journal of Geology, v. 58, no. 4, p. 430-487.
- Lowenstam, H.A., and E.P. DuBois, 1946, Marine Pool, Madison County, Illinois, a new type of reservoir in Illinois: Illinois State Geological Survey Report of Investigations 114, 30 p.
- McCracken, D.D., and W.S. Dorn, 1964, Numerical methods and Fortran programming: John Wiley and Sons, New York, 457 p.
- McGinnis, L.D., J.P. Kempton, and P.C. Heigold, 1963, Relationship of gravity anomalies to a drift-filled bedrock valley system in Northern Illinois: Illinois State Geological Survey Circular 354, 23 p.
- Nevin, C.M., and R.E. Sherrill, 1929, Studies in differential compaction: American Association of Petroleum Geologists Bulletin, v. 13, no. 1, p. 1-22.
- Nettleton, L.L., 1940, Geophysical prospecting for oil: McGraw-Hill Book Company, New York, 444 p.
- Telford, W. M., L.P. Geldart, R.E. Sheriff, and D. A. Keys, 1976, Applied geophysics: Cambridge University Press, New York, 860 p.
- Whitaker, S.T., 1988a, Ramp-platform model for Silurian pinnacle reef distribution in the Illinois basin: Oil and Gas Journal, v. 86, no. 18, p. 102-108.
- Whitaker, S.T., 1988b, Silurian pinnacle reef distribution in Illinois: model for hydrocarbon exploration: Illinois State Geological Survey Illinois Petroleum 130, 32 p.

APPENDIX A. TREND SURFACE ANALYSIS

The regional Bouguer gravity anomaly surface $g^*(x,y)$ can be represented by a power series in x and y,

$$g^*(x,y) = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + ...$$

Equation (2)

and is determined subject to Gauss's principle of least squares,

N

$$\Delta g = \sum_{i=1}^{N} [g(x_i, y_i - g^*(x_i, y_i))]^2 = \text{minimum},$$

$$i = 1$$

Equation (3)

where N is the number of discrete observations. The necessary conditions for Δg to be a minimum are given

$$\frac{\delta \Delta g}{\delta a_{j}} = 0 \qquad j=1,2,3...,n;$$

Equation (4)

where n is the number of coefficients in the power series $g^*(x, y)$.

These conditions provide n linear equations in n unknowns, the coefficients a_j . The solution of these equations provides the value of the coefficients a_j that in turn determine the desired regional Bouguer gravity anomaly surface, $g^*(x, y)$ (Coons et al., 1964).

APPENDIX B. SECOND VERTICAL DERIVATIVE

The second vertical derivative of Δg with respect to z can be calculated from the second horizontal partial derivative with respect to x and the second horizontal partial derivative of Δg with respect to y according to Laplace's equation,

$$\frac{\delta^2 \Delta g}{\delta x^2} + \frac{\delta^2 \Delta g}{\delta y^2} + \frac{\delta^2 \Delta g}{\delta z^2} = 0$$

Equation (5)

In this study a finite difference method was used to calculate and $\delta^2 \Delta g/\delta x^2$ and $\delta^2 \Delta g/\delta y^2$ (McCracken and Dorn, 1964). A grid with constant and equal nodal spacing in both the x and y directions was superimposed on the residual Bouguer gravity anomaly surface, g(x,y), and interpolated values of g(x,y) were determined at each node. Further,

$$\frac{\delta^2 \Delta g(I,J) = \underline{\Delta g(I+1,J) - 2\Delta g(I,J) + \Delta g(I-1,J)}}{\delta x^2} \qquad I = 2,3,...,M$$

Equation (6)

$$\frac{\delta^2 \Delta g}{\delta y^2} \left(\mathsf{I}, \mathsf{J} \right) = \frac{\Delta g \left(\mathsf{I}, \mathsf{J} + 1 \right) - 2 \Delta g \left(\mathsf{I}, \mathsf{J} \right) + \Delta g \left(\mathsf{I}, \mathsf{J} - 1 \right)}{\left(\Delta y \right)^2} \qquad \mathsf{J} = 2, 3, ..., \mathsf{N}$$

Equation (7)

where I = nodal index in the x-direction

M = number of nodes in the x-direction

J = nodal index in the y-direction

N = number of nodes in the y-direction

 $\Delta x = \Delta y = \text{nodal spacing}$

Another way to evaluate $\delta^2\Delta g/\delta z^2$ from equation 5 involves the use of the cubic spline interpolation applied to the rows and columns of the gridded residual Bouguer gravity anomaly data. Note that there is exactly one cubic polynomial

$$C(x) = \sum_{m=0}^{\infty} \alpha_{m(x-a)} m$$

Equation (8)

which assumes given values for C(x) and C'(x) at the end points of any interval [a, b] providing a does not equal b. The resultant polynomial is

$$C(x) = C(a) + C'(a)(x-a)$$

$$+ \left[\frac{3C(b) - C(a)}{(b-a)^2} - \frac{C'(b) + 2C'(a)}{b-a} \right] (x-a)^2$$

$$+ \left[\frac{-2C(b) - C(a)}{(b-a)^3} + \frac{C'(b) + C'(a)}{(b-a)^2} \right] (x-a)^3$$

Equation (9)

If C_i and C_i ' given for i=1,2,...,M, there exists exactly one piecewise cubic polynomial C(x) with nodes $x_1,x_2,...,x_m$ that satisfies $C(x^i)=C_i$ and $C'(x_i)=C_i$ ' (DeBoor, 1962).

The second derivative of the cubic polynomial above is

$$C''(x) = 2 \left[\frac{3C(b) - C(a)}{(b-a)^2} - \frac{C'(b) + 2C'(a)}{b-a} \right]$$

$$+ 6 \left[\frac{-2C(b) - C(a)}{(b-a)^3} + \frac{C'(b) + C'(a)}{(b-a)^2} \right] (x-a)$$

Equation (10)

This result can be used to calculate $\delta^2 \Delta g/\delta x^2$ and $\delta^2 \Delta g/\delta y^2$ at any point on or within the gridded residual Bouguer gravity anomaly values.